

Sets

An introduction of sets and its definition in mathematics. The concept of sets is used for the foundation of various topics in mathematics.

To learn sets we often talk about the collection of objects, such as a set of vowels, set of negative numbers, a group of friends, a list of fruits, a bunch of keys, etc.

What is set (in mathematics)?

The collection of well-defined distinct objects is known as a set. The word well-defined refers to a specific property which makes it easy to identify whether the given object belongs to the set or not. The word 'distinct' means that the objects of a set must be all different.

For example:

1. The collection of children in class VII whose weight exceeds 35 kg represents a set.
2. The collection of all the intelligent children in class VII does not represent a set because the word intelligent is vague. What may appear intelligent to one person may not appear the same to another person.

Elements of Set:

The different objects that form a set are called the elements of a set. The elements of the set are written in any order and are not repeated. Elements are denoted by small letters.

Notation of a Set:

A set is usually denoted by capital letters and elements are denoted by small letters

If x is an element of set A , then we say $x \in A$. [**x belongs to A**]

If x is not an element of set A , then we say $x \notin A$. [**x does not belong to A**]

For example:

The collection of vowels in the English alphabet.

Solution :

Let us denote the set by V , then the elements of the set are a, e, i, o, u or we can say, $V = [a, e, i, o, u]$.

We say $a \in V, e \in V, i \in V, o \in V$ and $u \in V$.

Also, we can say $b \notin V, c \notin V, d \notin V$, etc.

Elements of a Set

What are the elements of a set or members of a set?

The objects used to form a set are called its element or its members.

Generally, the elements of a set are written inside a pair of curly (idle) braces and are represented by commas. The name of the set is always written in capital letter.

Solved Examples to find the elements or members of a set:

1. $A = \{v, w, x, y, z\}$

Here 'A' is the name of the set whose elements (members) are v, w, x, y, z.

2. If a set $A = \{3, 6, 9, 10, 13, 18\}$. State whether the following statements are 'true' or 'false':

(i) $7 \in A$

(ii) $12 \notin A$

(iii) $13 \in A$

(iv) $9, 12 \in A$

(v) $12, 14, 15 \in A$

Solution:

(i) $7 \in A$

False, since the element 7 does not belongs to the given set A.

(ii) $10 \notin A$

False, since the element 10 belongs to the given set A.

(iii) $13 \in A$

True, since the element 13 belongs to the given set A.

(iv) $9, 10 \in A$

True, since the elements 9 and 12 both belong to the given set A.

(v) $10, 13, 14 \in A$

False, since the element 14 does not belongs to the given set A.

3. If set $Z = \{4, 6, 8, 10, 12, 14\}$. State which of the following statements are 'correct' and which are 'wrong' along with the correct explanations

(i) $5 \in Z$

(ii) $12 \in Z$

(iii) $14 \in Z$

(iv) $9 \in Z$

(v) Z is a set of even numbers between 2 and 16.

(vi) 4, 6 and 10 are members of the set Z.

Solution:

(i) $5 \in Z$

Wrong, since 5 does not belong to the given set Z i.e. $5 \notin Z$

(ii) $12 \in Z$

Correct, since 12 belongs to the given set Z.

(iii) $14 \in Z$

Correct, since 14 belongs to the given set Z.

(iv) $9 \in Z$

Wrong, since 9 does not belong to the given set Z i.e. $9 \notin Z$

(v) Z is a set of even numbers between 2 and 16.

Correct, since the elements of the set Z consist of all the multiples of 2 between 2 and 16.

(vi) 4, 6 and 10 are members of the set Z.

Correct, since the 4, 6 and 10 those numbers belong to the given set Z.

Properties of Sets

What are the two basic properties of sets?

The two basic properties to represent a set are explained below using various examples.

1. The change in order of writing the elements does not make any changes in the set.

In other words the order in which the elements of a set are written is not important. Thus, the set $\{a, b, c\}$ can also be written as $\{a, c, b\}$ or $\{b, c, a\}$ or $\{b, a, c\}$ or $\{c, a, b\}$ or $\{c, b, a\}$.

For Example:

Set $A = \{4, 6, 7, 8, 9\}$ is same as set $A = \{8, 4, 9, 7, 6\}$

i.e., $\{4, 6, 7, 8, 9\} = \{8, 4, 9, 7, 6\}$

Similarly, $\{w, x, y, z\} = \{x, z, w, y\} = \{z, w, x, y\}$ and so on.

2. If one or many elements of a set are repeated, the set remains the same.

In other words the elements of a set should be distinct. So, if any element of a set is repeated number of times in the set, we consider it as a single element. Thus, $\{1, 1, 2, 2, 3, 3, 4, 4, 4\} = \{1, 2, 3, 4\}$

The set of letters in the word 'GOOGLE' = $\{G, O, L, E\}$

For Example:

The set $A = \{5, 6, 7, 6, 8, 5, 9\}$ is same as set $A = \{5, 6, 7, 8, 9\}$

i.e., $\{5, 6, 7, 6, 8, 5, 9\} = \{5, 6, 7, 8, 9\}$

In general, the elements of a set are not repeated. Thus,

(i) if T is a set of letters of the word 'moon': then $T = \{m, o, n\}$,

There are two o's in the word 'moon' but it is written in the set only once.

(ii) if $U = \{\text{letters of the word 'COMMITTEE'}\}$; then $U = \{C, O, M, T, E\}$

Solved examples using the properties of sets:

1. Write the set of vowels used in the word 'UNIVERSITY'.

Solution:

Set $V = \{U, I, E\}$

2. For each statement, given below, state whether it is true or false along with the explanations.

(i) $\{9, 9, 9, 9, 9, \dots\dots\dots\} = \{9\}$

(ii) $\{p, q, r, s, t\} = \{t, s, r, q, p\}$

Solution:

(i) $\{9, 9, 9, 9, 9, \dots\dots\dots\} = \{9\}$

True, since repetition of elements does not change the set.

(ii) $\{p, q, r, s, t\} = \{t, s, r, q, p\}$

True, since the change in order of writing the elements does not change the set.

Standard Sets of Numbers

The standard sets of numbers can be expressed in all the three forms of representation of a set i.e., statement form, roster form, set builder form.

1. **N** = Natural numbers

= Set of all numbers starting from 1 → Statement form

= Set of all numbers 1, 2, 3,

= {1, 2, 3,} → Roster form

= {x : x is a counting number starting from 1} → Set builder form

Therefore, the set of natural numbers is denoted by **N** i.e., **N** = {1, 2, 3,}

2. **W** = Whole numbers

= Set containing zero and all natural numbers → Statement form

= {0, 1, 2, 3,} → Roster form

= {x : x is a zero and all natural numbers} → Set builder form

Therefore, the set of whole numbers is denoted by **W** i.e., **W** = {0, 1, 2,}

3. **Z** or **I** = Integers

= Set containing negative of natural numbers, zero and the natural numbers → Statement form

= {..., -3, -2, -1, 0, 1, 2, 3,} → Roster form

= {x : x is a containing negative of natural numbers, zero and the natural numbers} → Set builder form

Therefore, the set of integers is denoted by **I** or **Z** i.e., **I** = {..., -2, -1, 0, 1, 2,}

4. **E** = Even natural numbers.

= Set of natural numbers, which are divisible by 2 → Statement form

= {2, 4, 6, 8,} → Roster form

= {x : x is a natural number, which are divisible by 2} → Set builder form

Therefore, the set of even natural numbers is denoted by **E** i.e., **E** = {2, 4, 6, 8,.....}

5. **O** = Odd natural numbers.

= Set of natural numbers, which are not divisible by 2 → Statement form

= {1, 3, 5, 7, 9,} → Roster form

$= \{x : x \text{ is a natural number, which are not divisible by } 2\}$ $\rightarrow$ Set builder form

Therefore, the set of odd natural numbers is denoted by **O** i.e., **O** = {1, 3, 5, 7, 9,.....}

Therefore, almost every standard sets of numbers can be expressed in all the three methods as discussed above.

Cardinal Number of a Set

What is the cardinal number of a set?

The number of distinct elements in a finite set is called its cardinal number. It is denoted as $n(A)$ and read as ‘the number of elements of the set’.

For example:

(i) Set $A = \{2, 4, 5, 9, 15\}$ has 5 elements.

Therefore, the cardinal number of set $A = 5$. So, it is denoted as $n(A) = 5$.

(ii) Set $B = \{w, x, y, z\}$ has 4 elements.

Therefore, the cardinal number of set $B = 4$. So, it is denoted as $n(B) = 4$.

(iii) Set $C = \{\text{Florida, New York, California}\}$ has 3 elements.

Therefore, the cardinal number of set $C = 3$. So, it is denoted as $n(C) = 3$.

(iv) Set $D = \{3, 3, 5, 6, 7, 7, 9\}$ has 5 element.

Therefore, the cardinal number of set $D = 5$. So, it is denoted as $n(D) = 5$.

(v) Set $E = \{ \}$ has no element.

Therefore, the cardinal number of set $D = 0$. So, it is denoted as $n(D) = 0$.

Note:

(i) Cardinal number of an infinite set is not defined.

(ii) Cardinal number of empty set is 0 because it has no element.

Solved examples on Cardinal number of a set:

1. Write the cardinal number of each of the following sets:

(i) $X = \{\text{letters in the word MALAYALAM}\}$

(ii) $Y = \{5, 6, 6, 7, 11, 6, 13, 11, 8\}$

(iii) $Z = \{\text{natural numbers between 20 and 50, which are divisible by 7}\}$

Solution:

(i) Given, $X = \{\text{letters in the word MALAYALAM}\}$

Then, $X = \{M, A, L, Y\}$

Therefore, cardinal number of set $X = 4$, i.e., $n(X) = 4$

(ii) Given, $Y = \{5, 6, 6, 7, 11, 6, 13, 11, 8\}$

Then, $Y = \{5, 6, 7, 11, 13, 8\}$

Therefore, cardinal number of set $Y = 6$, i.e., $n(Y) = 6$

(iii) Given, $Z = \{\text{natural numbers between 20 and 50, which are divisible by 7}\}$

Then, $Z = \{21, 28, 35, 42, 49\}$

Therefore, cardinal number of set $Z = 5$, i.e., $n(Z) = 5$

2. Find the cardinal number of a set from each of the following:

(i) $P = \{x \mid x \in \mathbb{N} \text{ and } x^2$

$< 30\}$

(ii) $Q = \{x \mid x \text{ is a factor of } 20\}$

Solution:

(i) Given, $P = \{x \mid x \in \mathbb{N} \text{ and } x^2$

$< 30\}$

Then, $P = \{1, 2, 3, 4, 5\}$

Therefore, cardinal number of set $P = 5$, i.e., $n(P) = 5$

(ii) Given, $Q = \{x \mid x \text{ is a factor of } 20\}$

Then, $Q = \{1, 2, 4, 5, 10, 20\}$

Therefore, cardinal number of set $Q = 6$, i.e., $n(Q) = 6$

Representation of a Set

In representation of a set the following three methods are commonly used:

- (i) Statement form method
- (ii) Roster or tabular form method
- (iii) Rule or set builder form method

1. Statement form:

In this, well-defined description of the elements of the set is given and the same are enclosed in curly brackets.

For example:

- (i) The set of odd numbers less than 7 is written as: {odd numbers less than 7}.
- (ii) A set of football players with ages between 22 years to 30 years.
- (iii) A set of numbers greater than 30 and smaller than 55.
- (iv) A set of students in class VII whose weights are more than your weight.

2. Roster form or tabular form:

In this, elements of the set are listed within the pair of brackets { } and are separated by commas.

For example:

- (i) Let N denote the set of first five natural numbers.

Therefore, $N = \{1, 2, 3, 4, 5\}$ → Roster Form

- (ii) The set of all vowels of the English alphabet.

Therefore, $V = \{a, e, i, o, u\}$ → Roster Form

- (iii) The set of all odd numbers less than 9.

Therefore, $X = \{1, 3, 5, 7\}$ → Roster Form

- (iv) The set of all natural number which divide 12.

Therefore, $Y = \{1, 2, 3, 4, 6, 12\}$ → Roster Form

- (v) The set of all letters in the word MATHEMATICS.

Therefore, $Z = \{M, A, T, H, E, I, C, S\}$ → Roster Form

- (vi) W is the set of last four months of the year.

Therefore, $W = \{\text{September, October, November, December}\}$ → Roster Form

Note:

The order in which elements are listed is immaterial but elements must not be repeated.

3. Set builder form:

In this, a rule, or the formula or the statement is written within the pair of brackets so that the set is well defined. In the set builder form, all the elements of the set, must possess a single property to become the member of that set.

In this form of representation of a set, the element of the set is described by using a symbol 'x' or any other variable followed by a colon. The symbol ':' or '|' is used to denote such that and then we write the property possessed by the elements of the set and enclose the whole description in braces. In this, the colon stands for 'such that' and braces stand for 'set of all'.

For example:

(i) Let P is a set of counting numbers greater than 12;
the set P in set-builder form is written as :

$$P = \{x : x \text{ is a counting number and greater than } 12\}$$

or

$$P = \{x | x \text{ is a counting number and greater than } 12\}$$

This will be read as, 'P is the set of elements x such that x is a counting number and is greater than 12'.

Note:

The symbol ':' or '|' placed between 2 x's stands for such that.

(ii) Let A denote the set of even numbers between 6 and 14. It can be written in the set builder form as;

$$A = \{x | x \text{ is an even number, } 6 < x < 14\}$$

$$\text{or } A = \{x : x \in P, 6 < x < 14 \text{ and } P \text{ is an even number}\}$$

(iii) If $X = \{4, 5, 6, 7\}$. This is expressed in roster form.

Let us express in set builder form.

$$X = \{x : x \text{ is a natural number and } 3 < x < 8\}$$

(iv) The set A of all odd natural numbers can be written as

$$A = \{x : x \text{ is a natural number and } x = 2n + 1 \text{ for } n \in W\}$$

Solved example using the three methods of representation of a set:

The set of integers lying between -2 and 3.

Statement form: {I is a set of integers lying between -2 and 3}

Roster form: $I = \{-1, 0, 1, 2\}$

Set builder form: $I = \{x : x \in I, -2 < x < 3\}$

Types of Sets

What are the different types of sets?

The different types of sets are explained below with examples.

Empty Set or Null Set:

A set which does not contain any element is called an empty set, or the null set or the void set and it is denoted by $\emptyset$ and is read as phi. In roster form, $\emptyset$ is denoted by $\{\}$. An empty set is a finite set, since the number of elements in an empty set is finite, i.e., 0.

For example: (a) The set of whole numbers less than 0.

(b) Clearly there is no whole number less than 0.

Therefore, it is an empty set.

(c) $N = \{x : x \in N, 3 < x < 4\}$

- Let $A = \{x : 2 < x < 3, x \text{ is a natural number}\}$

Here A is an empty set because there is no natural number between 2 and 3.

- Let $B = \{x : x \text{ is a composite number less than } 4\}$.

Here B is an empty set because there is no composite number less than 4.

Note:

$\emptyset \neq \{0\} \therefore$ has no element.

$\{0\}$ is a set which has one element 0.

The cardinal number of an empty set, i.e., $n(\emptyset) = 0$

Singleton Set:

A set which contains only one element is called a singleton set.

For example:

- $A = \{x : x \text{ is neither prime nor composite}\}$

It is a singleton set containing one element, i.e., 1.

- $B = \{x : x \text{ is a whole number, } x < 1\}$

This set contains only one element 0 and is a singleton set.

- Let $A = \{x : x \in N \text{ and } x^2 = 4\}$

Here A is a singleton set because there is only one element 2 whose square is 4.

- Let $B = \{x : x \text{ is an even prime number}\}$

Here B is a singleton set because there is only one prime number which is even, i.e., 2.

Finite Set:

A set which contains a definite number of elements is called a finite set. Empty set is also called a finite set.

For example:

- The set of all colors in the rainbow.
- $N = \{x : x \in N, x < 7\}$
- $P = \{2, 3, 5, 7, 11, 13, 17, \dots, 97\}$

Infinite Set:

The set whose elements cannot be listed, i.e., set containing never-ending elements is called an infinite set.

For example:

- Set of all points in a plane
- $A = \{x : x \in N, x > 1\}$
- Set of all prime numbers
- $B = \{x : x \in W, x = 2n\}$

Note:

All infinite sets cannot be expressed in roster form.

For example:

The set of real numbers since the elements of this set do not follow any particular pattern.

Cardinal Number of a Set:

The number of distinct elements in a given set A is called the cardinal number of A. It is denoted by $n(A)$.

For example:

- $A = \{x : x \in N, x < 5\}$

$$A = \{1, 2, 3, 4\}$$

$$\text{Therefore, } n(A) = 4$$

- $B = \text{set of letters in the word ALGEBRA}$

$$B = \{A, L, G, E, B, R\}$$

$$\text{Therefore, } n(B) = 6$$

Equivalent Sets:

Two sets A and B are said to be equivalent if their cardinal number is same, i.e., $n(A) = n(B)$. The symbol for denoting an equivalent set is ' $\leftrightarrow$ '.

For example:

$A = \{1, 2, 3\}$ Here $n(A) = 3$

$B = \{p, q, r\}$ Here $n(B) = 3$

Therefore, $A \leftrightarrow B$

Equal sets:

Two sets A and B are said to be equal if they contain the same elements. Every element of A is an element of B and every element of B is an element of A.

For example:

$A = \{p, q, r, s\}$

$B = \{p, s, r, q\}$

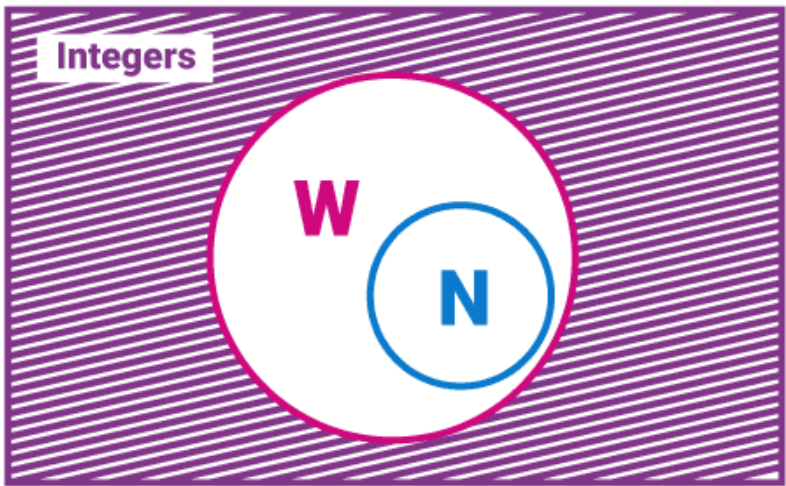
Therefore, $A = B$

The various types of sets and their definitions are explained above with the help of examples.

Venn Diagram Questions

Venn diagram questions with solutions are given here for students to practice various questions based on Venn diagrams. These questions are beneficial for both school examinations and competitive exams. Practising these questions will develop a skill to solve any problem on [Venn diagrams](#) quickly.

Venn diagrams were first introduced by John Venn to represent various propositions in a diagrammatic way. Venn diagrams are used for representing relationships between given sets. For example, natural numbers and whole numbers are subsets of integers represented by the Venn diagram:



Using Venn diagrams, we can easily understand whether given sets are subsets of each other or disjoint sets or have something in common.

Following are some set operations and their meaning useful while solving problems on the Venn diagram:

Set Operations	Meaning
$A \subset B$	Set A is a proper subset of B, or A is contained in B.
$A \cup B$	Set of all those elements which either belong to A or belong to B
$A \cap B$	Set of all those elements which belong to both A and B
A^C or A'	Set of all those elements which are not in A
$A - B$	Set of all those elements which only belong to A
$A \ominus B$	Symmetric difference: Set of all those elements which either belong to A or belong to B, but not in both .

Some important formulae:

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$ $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$ $A \cup U = U$; U is universal set $A \cap U = A$; U is universal set $A^C = U - A$ $(A^C)^C = A$ $A \cup A' = U$ $A \cap A' = \phi$ $(A \cup B)' = A' \cap B'$ $(A \cap B)' = A' \cup B'$ $U' = \phi$ $\phi' = U$

Venn diagram Questions with Solution

Let us practice some questions based on Venn diagrams.

Question 1: If A and B are two sets such that number of elements in A is 24, number of elements in B is 22 and number of elements in both A and B is 8, find:

(i) $n(A \cup B)$

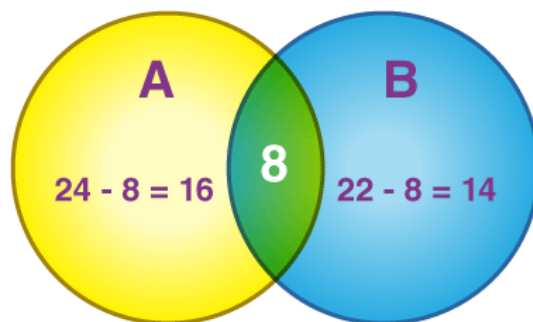
(ii) $n(A - B)$

(ii) $n(B - A)$

Solution:

Given, $n(A) = 24$, $n(B) = 22$ and $n(A \cap B) = 8$

The Venn diagram for the given information is:



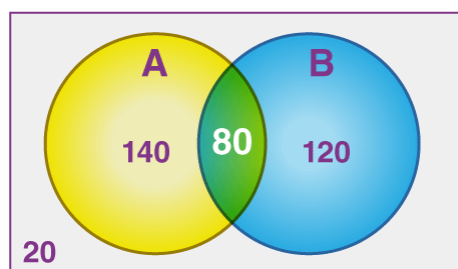
(i) $n(A \cup B) = n(A) + n(B) - n(A \cap B) = 24 + 22 - 8 = 38$.

(ii) $n(A - B) = n(A) - n(A \cap B) = 24 - 8 = 16$.

(iii) $n(B - A) = n(B) - n(A \cap B) = 22 - 8 = 14$.

Question 2: According to the survey made among 200 students, 140 students like cold drinks, 120 students like milkshakes and 80 like both. How many students like atleast one of the drinks?

Solution:



Number of students like cold drinks = $n(A) = 140$

Number of students like milkshake = $n(B) = 120$

Number of students like both = $n(A \cap B) = 80$

Number of students like atleast one of the drinks = $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$$= 140 + 120 - 80$$

$$= 180.$$

Question 3: In a group of 500 people, 350 people can speak English, and 400 people can speak Hindi. Find how many people can speak both languages?

Solution:

Let H be the set of people who can speak Hindi and E be the set of people who can speak English. Then,

$$n(H) = 400$$

$$n(E) = 350$$

$$n(H \cup E) = 500$$

We have to find $n(H \cap E)$.

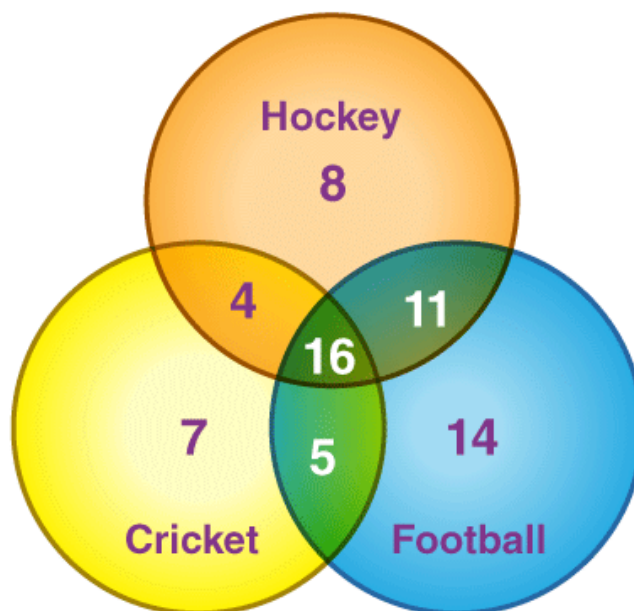
$$\text{Now, } n(H \cup E) = n(H) + n(E) - n(H \cap E)$$

$$\Rightarrow 500 = 400 + 350 - n(H \cap E)$$

$$\Rightarrow n(H \cap E) = 750 - 500 = 250.$$

$\therefore$ 250 people can speak both languages.

Questions 4: The following Venn diagram shows games played by the number of students in a class:



How many students like only cricket and only football?

Solution:

As per the given Venn diagram,

Number of students only like cricket = 7

Number of students only like football = 14

$\therefore$ Number of students like only cricket and only football = $7 + 14 = 21$.

Question 5: In a class of 40 students, 20 have chosen Mathematics, 15 have chosen mathematics but not biology. If every student has chosen either mathematics or biology or both, find the number of students who chose both mathematics and biology and the number of students chose biology but not mathematics.

Solution:

Let, $M \equiv$ Set of students who chose mathematics

$B \equiv$ Set of students who chose biology

$$n(M \cup B) = 40$$

$$n(M) = 20$$

$$n(B) = n(M \cup B) - n(M)$$

$$\Rightarrow n(B) = 40 - 20 = 20$$

$$n(M - B) = 15$$

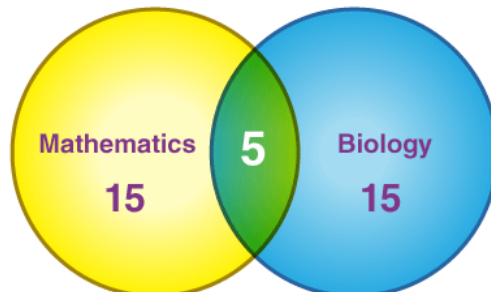
$$n(M) = n(M - B) + n(M \cap B)$$

$$\Rightarrow 20 = 15 + n(M \cap B)$$

$$\Rightarrow n(M \cap B) = 20 - 15 = 5$$

$$n(B - M) = n(B) - n(M \cap B)$$

$$\Rightarrow n(B - M) = 20 - 5 = 15$$



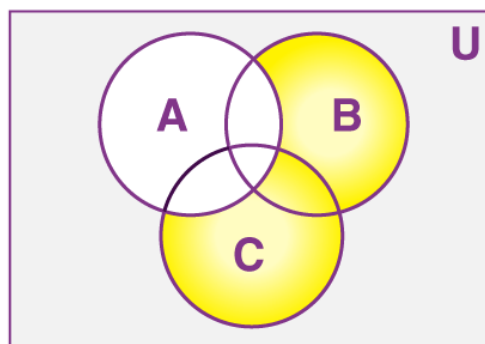
Question 6: Represent The following as Venn diagram:

(i) $A' \cap (B \cup C)$

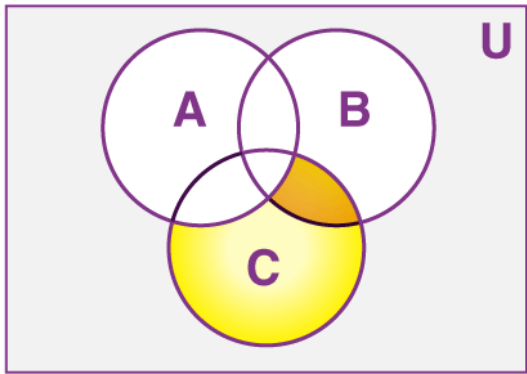
(ii) $A' \cap (C - B)$

Solution:

(i)



(ii)



Question 7: In a survey among 140 students, 60 likes to play videogames, 70 likes to play indoor games, 75 likes to play outdoor games, 30 play indoor and outdoor games, 18 like to play video games and outdoor games, 42 play video games and indoor games and 8 likes to play all types of games. Use the Venn diagram to find

(i) Students who play only outdoor games

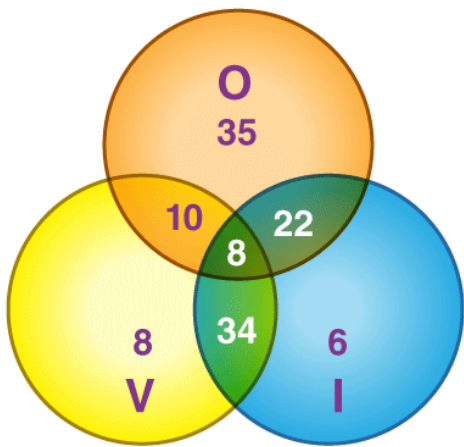
(ii) Students who play video games and indoor games, but not outdoor games.

Solution:

Let $V \equiv$ Play video games

$I \equiv$ Play indoor games

$O \equiv$ Play outdoor games



$$n(V) = 60, n(I) = 70, n(O) = 75$$

$$n(I \cap O) = 30, n(V \cap O) = 18, n(V \cap I) = 42$$

$$n(V \cap I \cap O) = 8$$

Hence, by Venn diagram

Number of students only like to play only outdoor games = 35

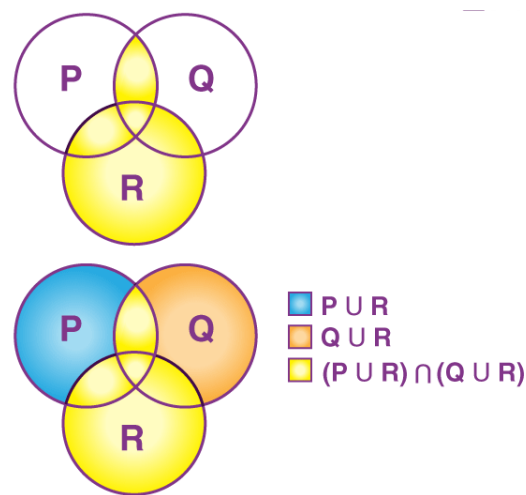
Number of students like to play video games and indoor games but not outdoor games = 34

Note: Always begin to fill the Venn diagram from the innermost part.

Question 8: Using the Venn diagrams, verify $(P \cap Q) \cup R = (P \cup R) \cap (Q \cup R)$.

Solution:

The shaded portion represents $(P \cap Q) \cup R$ in the Venn diagram.

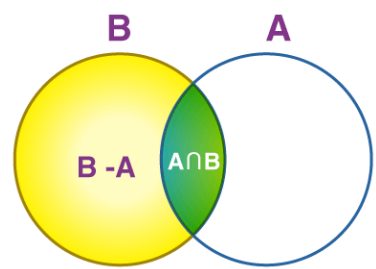


Comparing both the shaded portion in both the Venn diagram, we get $(P \cap Q) \cup R = (P \cup R) \cap (Q \cup R)$.

Question 9: Prove using the Venn diagram: $(B - A) \cup (A \cap B) = B$.

Solution:

From the Venn diagram, it is clear that $(B - A) \cup (A \cap B) = B$



Question 10: In a survey, it is found that 21 people read English newspaper, 26 people read Hindi newspaper, and 29 people read regional language newspaper. If 14 people read both English and Hindi newspapers; 15 people read both Hindi and regional language newspapers; 12 people read both English and regional language newspaper and 8 read all types of newspapers, find:

- (i) How many people were surveyed?
- (ii) How many people read only regional language newspapers?

Solution:

Let $A \equiv$ People who read English newspapers.

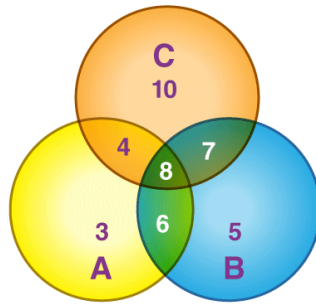
$B \equiv$ People who read Hindi newspapers.

$C \equiv$ People who read Hindi newspapers.

$$n(A) = 21, n(B) = 26, n(C) = 29$$

$$n(A \cap B) = 14, n(B \cap C) = 15, n(A \cap C) = 12$$

$$n(A \cap B \cap C) = 8$$



(i) Number of people surveyed = $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C) = 21 + 26 + 29 - 14 - 15 - 12 + 8 = 43$

(ii) By the Venn diagram, number of people who only read regional language newspapers = 10.

Venn Diagram Examples, Problems and Solutions

The best way to explain how the Venn diagram works and what its formulas show is to give 2 or 3 circles **Venn diagram examples** and problems with solutions.

Problem-solving using Venn diagram is a widely used approach in many areas such as statistics, data science, business, set theory, math, logic and etc.

On this page:

- What is Venn diagram? Definition and meaning.
- Venn diagram formula with an explanation.
- Examples of 2 and 3 sets Venn diagrams: practice problems with solutions, questions, and answers.
- Simple 4 circles Venn diagram with word problems.
- Compare and contrast Venn diagram example.

Let's define it:

A Venn Diagram is an illustration that shows **logical relationships** between two or more sets (grouping items). Venn diagram uses circles (both overlapping and nonoverlapping) or other shapes.

Commonly, Venn diagrams show how given items are similar and different.

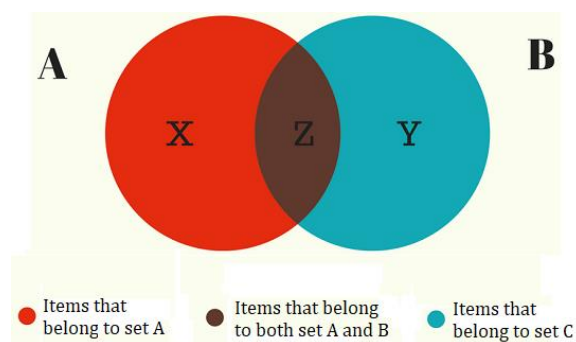
Despite Venn diagram with 2 or 3 circles are the most common type, there are also many diagrams with a larger number of circles (5,6,7,8,10...). Theoretically, they can have unlimited circles.

Venn Diagram General Formula

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

Don't worry, there is no need to remember this formula, once you grasp the meaning. Let's see the explanation with an example.

This is a very simple Venn diagram example that shows the relationship between two overlapping sets X, Y.



Where:

X – the number of items that belong to set A

Y – the number of items that belong to set B

Z – the number of items that belong to set A and B both

From the above Venn diagram, it is quite clear that

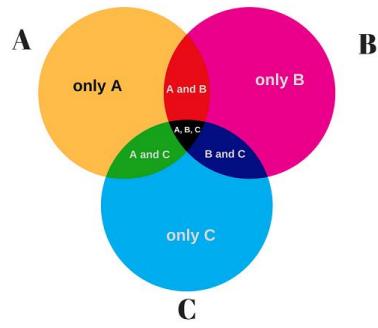
$n(A) = x + z$

$n(B) = y + z$

$n(A \cap B) = z$

$n(A \cup B) = x + y + z$.

Now, let’s move forward and think about Venn Diagrams with 3 circles.



Following the same logic, we can write **the formula for 3 circles Venn diagram**:

$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$

Venn Diagram Examples (Problems with Solutions)

As we already know how the Venn diagram works, we are going to give some practical examples (problems with solutions) from the real life.

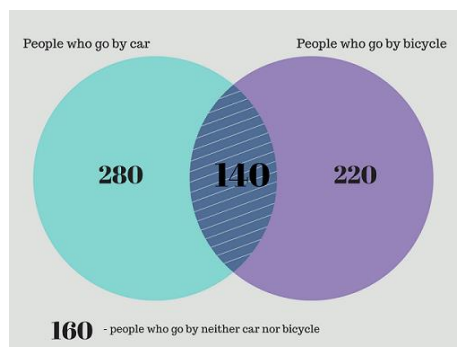
2 Circle Venn Diagram Examples (word problems):

Suppose that in a town, 800 people are selected by random [types of sampling methods](#). 280 go to work by car only, 220 go to work by bicycle only and 140 use both ways – sometimes go with a car and sometimes with a bicycle.

Here are some important questions we will find the answers:

- How many people go to work by car only?
- How many people go to work by bicycle only?
- How many people go by neither car nor bicycle?
- How many people use at least one of both transportation types?
- How many people use only one of car or bicycle?

The following Venn diagram represents the data above:



Now, we are going to answer our questions:

- Number of people who go to work by car only = 280
- Number of people who go to work by bicycle only = 220
- Number of people who go by neither car nor bicycle = 160

- Number of people who use at least one of both transportation types = $n(\text{only car}) + n(\text{only bicycle}) + n(\text{both car and bicycle}) = 280 + 220 + 140 = 640$
- Number of people who use only one of car or bicycle = $280 + 220 = 500$

Note: The number of people who go by neither car nor bicycle (160) is illustrated outside of the circles. It is a common practice the number of items that belong to none of the studied sets, to be illustrated outside of the diagram circles.

We will deep further with a more complicated triple Venn diagram example.

3 Circle Venn Diagram Examples:

For the purposes of a [marketing research](#), a survey of 1000 women is conducted in a town. The results show that 52 % liked watching comedies, 45% liked watching fantasy movies and 60% liked watching romantic movies. In addition, 25% liked watching comedy and fantasy both, 28% liked watching romantic and fantasy both and 30% liked watching comedy and romantic movies both. 6% liked watching none of these movie genres.

Here are our questions we should find the answer:

- How many women like watching all the three movie genres?
- Find the number of women who like watching only one of the three genres.
- Find the number of women who like watching at least two of the given genres.

Let’s represent the data above in a more digestible way using the Venn diagram formula elements:

- **n(C)** = percentage of women who like watching comedy = 52%
- **n(F)** = percentage of women who like watching fantasy = 45%
- **n(R)**= percentage of women who like watching romantic movies= 60%
- **n(C∩F)** = 25%; **n(F∩R)** = 28%; **n(C∩R)** = 30%
- Since 6% like watching none of the given genres so, **n (C ∪ F ∪ R)** = 94%.

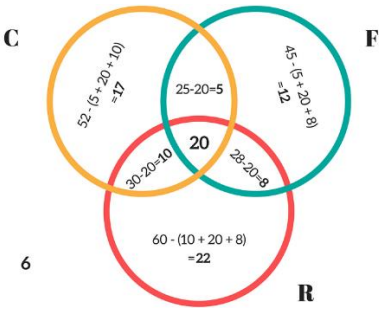
Now, we are going to apply the **Venn diagram formula for 3 circles**.

$$94\% = 52\% + 45\% + 60\% - 25\% - 28\% - 30\% + n(C \cap F \cap R)$$

Solving this simple math equation, lead us to:

$$n(C \cap F \cap R) = 20\%$$

It is a great time to make our Venn diagram related to the above situation (problem):



See, the Venn diagram makes our situation much more clear!

From the Venn diagram example, we can answer our questions with ease.

- The number of women who like watching all the three genres = 20% of 1000 = 200.
- Number of women who like watching only one of the three genres = (17% + 12% + 22%) of 1000 = 510
- The number of women who like watching **at least** two of the given genres = (number of women who like watching **only two** of the genres) +(number of women who like watching **all the three** genres) = (10 + 5 + 8 + 20)% i.e. 43% of 1000 = 430.

As we mentioned above 2 and 3 circle diagrams are much more common for problem-solving in many areas such as business, statistics, data science and etc. However, 4 circle Venn diagram also has its place.

4 Circles Venn Diagram Example:

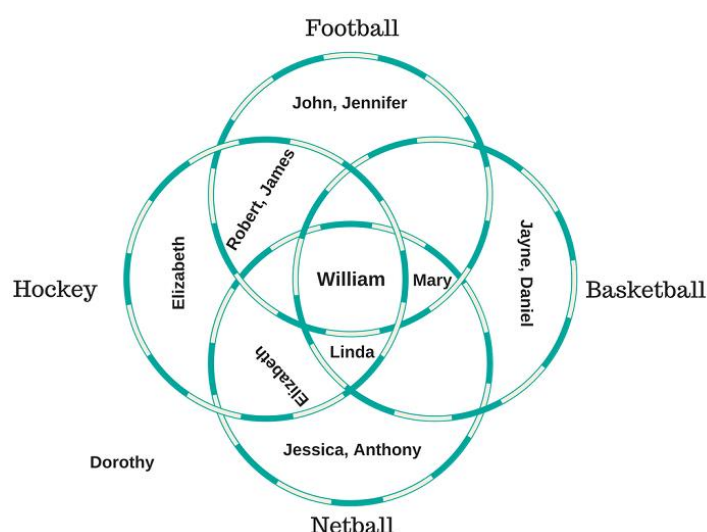
A set of students were asked to tell which sports they played in school.

The options are: Football, Hockey, Basketball, and Netball.

Here is the list of the results:

Sport	Name
Football	Robert, James, John, Mary, Jennifer, William
Hockey	Robert, William, Linda, Elizabeth, James
Basketball	William, Jayne, Linda, Daniel, Mary
Netball	Jessica, William, Linda, Elizabeth, Anthony, Mary
None	Dorothy

The next step is to draw a Venn diagram to show the data sets we have.



It is very clear who plays which sports. As you see the diagram also include the student who does not play any sports (Dorothy) by putting her name outside of the 4 circles.

From the above Venn diagram examples, it is obvious that this graphical tool can help you a lot in representing a variety of data sets. Venn diagram also is among the most popular [types of graphs](#) for identifying **similarities and differences**.

Word Problems on Sets

Word problems on sets are solved here to get the basic ideas how to use the properties of union and intersection of sets.

Solved basic word problems on sets:

1. Let A and B be two finite sets such that $n(A) = 20$, $n(B) = 28$ and $n(A \cup B) = 36$, find $n(A \cap B)$.

Solution:

Using the formula $n(A \cup B) = n(A) + n(B) - n(A \cap B)$.

then $n(A \cap B) = n(A) + n(B) - n(A \cup B)$

$$= 20 + 28 - 36$$

$$= 48 - 36$$

$$= 12$$

2. If $n(A - B) = 18$, $n(A \cup B) = 70$ and $n(A \cap B) = 25$, then find $n(B)$.

Solution:

Using the formula $n(A \cup B) = n(A - B) + n(A \cap B) + n(B - A)$

$$70 = 18 + 25 + n(B - A)$$

$$70 = 43 + n(B - A)$$

$$n(B - A) = 70 - 43$$

$$n(B - A) = 27$$

Now $n(B) = n(A \cap B) + n(B - A)$

$$= 25 + 27$$

$$= 52$$

Different types on word problems on sets:

3. In a group of 60 people, 27 like cold drinks and 42 like hot drinks and each person likes at least one of the two drinks. How many like both coffee and tea?

Solution:

Let A = Set of people who like cold drinks.

B = Set of people who like hot drinks.

Given

$$(A \cup B) = 60 \quad n(A) = 27 \quad n(B) = 42 \text{ then;}$$

$$n(A \cap B) = n(A) + n(B) - n(A \cup B)$$

$$= 27 + 42 - 60$$

$$= 69 - 60 = 9$$

$$= 9$$

Therefore, 9 people like both tea and coffee.

4. There are 35 students in art class and 57 students in dance class. Find the number of students who are either in art class or in dance class.

- When two classes meet at different hours and 12 students are enrolled in both activities.
- When two classes meet at the same hour.

Solution:

$$n(A) = 35, \quad n(B) = 57, \quad n(A \cap B) = 12$$

(Let A be the set of students in art class.

B be the set of students in dance class.)

(i) When 2 classes meet at different hours $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$$= 35 + 57 - 12$$

$$= 92 - 12$$

$$= 80$$

(ii) When two classes meet at the same hour, $A \cap B = \emptyset$ $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$$= n(A) + n(B)$$

$$= 35 + 57$$

$$= 92$$

Further concept to solve word problems on sets:

5. In a group of 100 persons, 72 people can speak English and 43 can speak French. How many can speak English only? How many can speak French only and how many can speak both English and French?

Solution:

Let A be the set of people who speak English.

B be the set of people who speak French.

A - B be the set of people who speak English and not French.

B - A be the set of people who speak French and not English.

$A \cap B$ be the set of people who speak both French and English.

Given,

$$n(A) = 72 \quad n(B) = 43 \quad n(A \cup B) = 100$$

$$\text{Now, } n(A \cap B) = n(A) + n(B) - n(A \cup B)$$

$$= 72 + 43 - 100$$

$$= 115 - 100$$

$$= 15$$

Therefore, Number of persons who speak both French and English = 15

$$n(A) = n(A - B) + n(A \cap B)$$

$$\Rightarrow n(A - B) = n(A) - n(A \cap B)$$

$$= 72 - 15$$

$$= 57$$

$$\text{and } n(B - A) = n(B) - n(A \cap B)$$

$$= 43 - 15$$

$$= 28$$

Therefore, Number of people speaking English only = 57

Number of people speaking French only = 28

Word problems on sets using the different properties (Union & Intersection):

6. In a competition, a school awarded medals in different categories. 36 medals in dance, 12 medals in dramatics and 18 medals in music. If these medals went to a total of 45 persons and only 4 persons got medals in all the three categories, how many received medals in exactly two of these categories?

Solution:

Let A = set of persons who got medals in dance.

B = set of persons who got medals in dramatics.

C = set of persons who got medals in music.

Given,

$$n(A) = 36 \qquad n(B) = 12 \qquad n(C) = 18$$

$$n(A \cup B \cup C) = 45 \qquad n(A \cap B \cap C) = 4$$

We know that number of elements belonging to exactly two of the three sets A, B, C

$$= n(A \cap B) + n(B \cap C) + n(A \cap C) - 3n(A \cap B \cap C)$$

$$= n(A \cap B) + n(B \cap C) + n(A \cap C) - 3 \times 4 \quad \dots\dots\dots(i)$$

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$$

$$\text{Therefore, } n(A \cap B) + n(B \cap C) + n(A \cap C) = n(A) + n(B) + n(C) + n(A \cap B \cap C) - n(A \cup B \cup C)$$

From (i) required number

$$= n(A) + n(B) + n(C) + n(A \cap B \cap C) - n(A \cup B \cup C) - 12$$

$$= 36 + 12 + 18 + 4 - 45 - 12$$

$$= 70 - 57$$

$$= 13$$

Apply set operations to solve the word problems on sets:

7. Each student in a class of 40 plays at least one indoor game chess, carrom and scrabble. 18 play chess, 20 play scrabble and 27 play carrom. 7 play chess and scrabble, 12 play scrabble and carrom and 4 play chess, carrom and scrabble. Find the number of students who play (i) chess and carrom. (ii) chess, carrom but not scrabble.

Solution:

Let A be the set of students who play chess

B be the set of students who play scrabble

C be the set of students who play carrom

Therefore, We are given $n(A \cup B \cup C) = 40$,

$$n(A) = 18, \quad n(B) = 20 \quad n(C) = 27,$$

$$n(A \cap B) = 7, \quad n(C \cap B) = 12 \quad n(A \cap B \cap C) = 4$$

We have

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$$

$$\text{Therefore, } 40 = 18 + 20 + 27 - 7 - 12 - n(C \cap A) + 4$$

$$40 = 69 - 19 - n(C \cap A)$$

$$40 = 50 - n(C \cap A) \quad n(C \cap A) = 50 - 40$$

$$n(C \cap A) = 10$$

Therefore, Number of students who play chess and carrom are 10.

Also, number of students who play chess, carrom and not scrabble.

$$= n(C \cap A) - n(A \cap B \cap C)$$

$$= 10 - 4$$

$$= 6$$

Therefore, we learned how to solve different types of word problems on sets without using Venn diagram.

Problems on Union of Sets

Solved problems on union of sets are given below to get a fair idea how to find the union of two or more sets.

We know, the union of two or more sets is a set which contains all the elements in those sets.

Solved problems on union of sets:

1. Let $A = \{x : x \text{ is a natural number and a factor of } 18\}$ and $B = \{x : x \text{ is a natural number and less than } 6\}$. Find $A \cup B$.

Solution:

$$A = \{1, 2, 3, 6, 9, 18\}$$

$$B = \{1, 2, 3, 4, 5\}$$

$$\text{Therefore, } A \cup B = \{1, 2, 3, 4, 5, 6, 9, 18\}$$

2. Let $A = \{0, 1, 2, 3, 4, 5\}$, $B = \{2, 4, 6, 8\}$ and $C = \{1, 3, 5, 7\}$

$$\text{Verify } (A \cup B) \cup C = A \cup (B \cup C)$$

Solution:

$$(A \cup B) \cup C = A \cup (B \cup C)$$

$$\text{L.H.S.} = (A \cup B) \cup C$$

$$A \cup B = \{0, 1, 2, 3, 4, 5, 6, 8\}$$

$$(A \cup B) \cup C = \{0, 1, 2, 3, 4, 5, 6, 7, 8\} \quad \dots\dots\dots (1)$$

$$\text{R.H.S.} = A \cup (B \cup C)$$

$$B \cup C = \{1, 2, 3, 4, 5, 6, 7, 8\}$$

$$A \cup (B \cup C) = \{0, 1, 2, 3, 4, 5, 6, 7, 8\} \quad \dots\dots\dots (2)$$

Therefore, from (1) and (2), we conclude that;

$$(A \cup B) \cup C = A \cup (B \cup C) \quad [\text{verified}]$$

More worked-out problems on union of sets to **find the union of three sets**.

3. Let $X = \{1, 2, 3, 4\}$, $Y = \{2, 3, 5\}$ and $Z = \{4, 5, 6\}$.

(i) Verify $X \cup Y = Y \cup X$

(ii) Verify $(X \cup Y) \cup Z = X \cup (Y \cup Z)$

Solution:

(i) $X \cup Y = Y \cup X$

$$\text{L.H.S} = X \cup Y$$

$$= \{1, 2, 3, 4\} \cup \{2, 3, 4\} = \{1, 2, 3, 4, 5\}$$

R.H.S. = $Y \cup X$

= $\{2, 3, 5\} \cup \{1, 2, 3, 4\} = \{2, 3, 5, 1, 4\}$

Therefore, $X \cup Y = Y \cup X$ [*verified*]

(ii) $(X \cup Y) \cup Z = X \cup (Y \cup Z)$

L.H.S. = $(X \cup Y) \cup Z$

$X \cup Y = \{1, 2, 3, 4\} \cup \{2, 3, 5\}$

= $\{1, 2, 3, 4, 5\}$

Now $(X \cup Y) \cup Z$

= $\{1, 2, 3, 4, 5, 6\} \cup \{4, 5, 6\}$

= $\{1, 2, 3, 4, 5, 6\}$

R.H.S. = $X \cup (Y \cup Z)$

$Y \cup Z = \{2, 3, 5\} \cup \{4, 5, 6\}$

= $\{2, 3, 4, 5, 6\}$

$X \cup (Y \cup Z) = \{1, 2, 3, 4\} \cup \{2, 3, 4, 5, 6\}$

Therefore, $(X \cup Y) \cup Z = X \cup (Y \cup Z)$ [*verified*]

Pair Factors of 18

The numbers which are multiplied together resulting in 18 is called the pair factor of 18. As discussed above, the pair factors of 18 can be positive or negative. Thus, the positive and negative pair factors of 18 are given below:

Positive Pair Factors of 18:

Positive Factors of 18	Positive Pair Factors of 18
1×18	$(1, 18)$
2×9	$(2, 9)$
3×6	$(3, 6)$

Negative Pair Factor of 18:

Negative Factors of 18	Negative Pair Factors of 18
-1×-18	$(-1, -18)$
-2×-9	$(-2, -9)$
-3×-6	$(-3, -6)$

Factors of 18 by Division Method

In the division method, the factors of 18 are found by dividing 18 by different integers. If the integers divide 18 exactly and leave the remainder zero, then those integers are the factors of 18. Now, let us start dividing 18 by 1 and proceed with the different integers.

- $18/1 = 18$ (Factor is 1 and Remainder is 0)
- $18/2 = 9$ (Factor is 2 and Remainder is 0)
- $18/3 = 6$ (Factor is 3 and Remainder is 0)
- $18/6 = 3$ (Factor is 6 and Remainder is 0)
- $18/9 = 2$ (Factor is 9 and Remainder is 0)
- $18/18 = 1$ (Factor is 18 and Remainder is 0)

If we divide 18 by any numbers other than 1, 2, 3, 6, 9 and 18, it leaves a remainder. Thus, the factors of 18 are 1, 2, 3, 6, 9 and 18.

Example 1:

Find the common factors of 18 and 17.

Solution:

The factors of 18 are 1, 2, 3, 6, 9 and 18.

The factors of 17 are 1 and 17.

As 17 is a prime number, the common factor of 18 and 17 is 1.

Example 2:

Find the common factors of 18 and 19.

Solution:

Factors of 18 = 1, 2, 3, 6, 9 and 18.

Factors of 19 = 1 and 19.

The common factor of 18 and 19 is 1, as 19 is a prime number.

Example 3:

Find the common factors of 18 and 9.

Solution:

The factors of 18 are 1, 2, 3, 6, 9 and 18.

The factors of 9 are 1, 3 and 9.

Therefore, the common factors of 18 and 9 are 1, 3 and 9.

Problems on Intersection of Sets

Solved problems on intersection of sets are given below to get a fair idea how to find the intersection of two or more sets.

We know, the intersection of two or more sets is a set which contains all the elements that are common in those sets.

Solved problems on intersection of sets:

1. Let $A = \{x : x \text{ is a natural number and a factor of } 18\}$

$B = \{x : x \text{ is a natural number and less than } 6\}$

Find $A \cup B$ and $A \cap B$.

Solution:

$A = \{1, 2, 3, 6, 9, 18\}$

$B = \{1, 2, 3, 4, 5\}$

Therefore, $A \cap B = \{1, 2, 3\}$

2. If $P = \{\text{multiples of } 3 \text{ between } 1 \text{ and } 20\}$ and $Q = \{\text{even natural numbers upto } 15\}$. Find the intersection of the two given set P and set Q.

Solution:

$P = \{\text{multiples of } 3 \text{ between } 1 \text{ and } 20\}$

So, $P = \{3, 6, 9, 12, 15, 18\}$

$Q = \{\text{even natural numbers upto } 15\}$

So, $Q = \{2, 4, 6, 8, 10, 12, 14\}$

Therefore, intersection of P and Q is the largest set containing only those elements which are common to both the given sets P and Q

Hence, $P \cap Q = \{6, 12\}$.

More worked-out problems on union of sets to **find the intersection of three sets**.

3. Let $A = \{0, 1, 2, 3, 4, 5\}$, $B = \{2, 4, 6, 8\}$ and $C = \{1, 3, 5, 7\}$

Verify $(A \cap B) \cap C = A \cap (B \cap C)$

Solution:

$(A \cap B) \cap C = A \cap (B \cap C)$

L.H.S. = $(A \cap B) \cap C$

$A \cap B = \{2, 4\}$

$(A \cap B) \cap C = \{\emptyset\}$ (1)

R.H.S. = $A \cap (B \cap C)$

$B \cap C = \{\emptyset\}$

$A \cap \{B \cap C\} = \{\emptyset\}$ (2)

Therefore, from (1) and (2), we conclude that;

$(A \cap B) \cap C = A \cap (B \cap C)$ [*verified*]

Difference of Two Sets

How to find the difference of two sets?

If A and B are two sets, then their difference is given by $A - B$ or $B - A$.

- If $A = \{2, 3, 4\}$ and $B = \{4, 5, 6\}$

$A - B$ means elements of A which are not the elements of B.

i.e., in the above example $A - B = \{2, 3\}$

In general, $B - A = \{x : x \in B, \text{ and } x \notin A\}$

- If A and B are disjoint sets, then $A - B = A$ and $B - A = B$

Solved examples to find the difference of two sets:

1. $A = \{1, 2, 3\}$ and $B = \{4, 5, 6\}$.

Find the difference between the two sets:

(i) A and B

(ii) B and A

Solution:

The two sets are disjoint as they do not have any elements in common.

(i) $A - B = \{1, 2, 3\} = A$

(ii) $B - A = \{4, 5, 6\} = B$

2. Let $A = \{a, b, c, d, e, f\}$ and $B = \{b, d, f, g\}$.

Find the difference between the two sets:

(i) A and B

(ii) B and A

Solution:

(i) $A - B = \{a, c, e\}$

Therefore, the elements a, c, e belong to A but not to B

(ii) $B - A = \{g\}$

Therefore, the element g belongs to B but not A.

3. Given three sets P, Q and R such that:

$P = \{x : x \text{ is a natural number between } 10 \text{ and } 16\}$,

$Q = \{y : y \text{ is a even number between } 8 \text{ and } 20\}$ and

$R = \{7, 9, 11, 14, 18, 20\}$

(i) Find the difference of two sets P and Q

(ii) Find $Q - R$

(iii) Find $R - P$

(iv) Find $Q - P$

Solution:

According to the given statements:

$$P = \{11, 12, 13, 14, 15\}$$

$$Q = \{10, 12, 14, 16, 18\}$$

$$R = \{7, 9, 11, 14, 18, 20\}$$

(i) $P - Q = \{\text{Those elements of set P which are not in set Q}\}$

$$= \{11, 13, 15\}$$

(ii) $Q - R = \{\text{Those elements of set Q not belonging to set R}\}$

$$= \{10, 12, 16\}$$

(iii) $R - P = \{\text{Those elements of set R which are not in set P}\}$

$$= \{7, 9, 18, 20\}$$

(iv) $Q - P = \{\text{Those elements of set Q not belonging to set P}\}$

$$= \{10, 16, 18\}$$

Complement of a Set

In complement of a set if ξ be the universal set and A a subset of ξ , then the complement of A is the set of all elements of ξ which are not the elements of A .

Symbolically, we denote the complement of A with respect to ξ as A' .

For Example; If $\xi = \{1, 2, 3, 4, 5, 6, 7\}$

$A = \{1, 3, 7\}$ find A' .

Solution:

We observe that 2, 4, 5, 6 are the only elements of ξ which do not belong to A .

Therefore, $A' = \{2, 4, 5, 6\}$

Note:

The complement of a universal set is an empty set.

The complement of an empty set is a universal set.

The set and its complement are disjoint sets.

For Example;

1. Let the set of natural numbers be the universal set and A is a set of even natural numbers,

then $A' = \{x: x \text{ is a set of odd natural numbers}\}$

2. Let $\xi =$ The set of letters in the English alphabet.

$A =$ The set of consonants in the English alphabet

then $A' =$ The set of vowels in the English alphabet.

3. Show that;

(a) The complement of a universal set is an empty set.

Let ξ denote the universal set, then

$\xi' =$ The set of those elements which are not in ξ .

$=$ empty set $= \phi$

Therefore, $\xi' = \phi$ so the complement of a universal set is an empty set.

(b) A set and its complement are disjoint sets.

Let A be any set then $A' =$ set of those elements of ξ which are not in A .

Let $x \notin A$, then x is an element of ξ not contained in A

So $x \notin A'$

Therefore, A and A' are disjoint sets.

Therefore, Set and its complement are disjoint sets

Similarly, in complement of a set when U be the universal set and A is a subset of U . Then the complement of A is the set all elements of U which are not the elements of A .

Symbolically, we write A' to denote the complement of A with respect to U .

Thus, $A' = \{x : x \in U \text{ and } x \notin A\}$

Obviously $A' = \{U - A\}$

For Example; Let $U = \{2, 4, 6, 8, 10, 12, 14, 16\}$

$A = \{6, 10, 4, 16\}$

$A' = \{2, 8, 12, 14\}$

We observe that 2, 8, 12, 14 are the only elements of U which do not belong to A .

Some properties of complement sets

(i) $A \cup A' = A' \cup A = U$ (Complement law)

(ii) $(A \cap B)' = \phi$ (Complement law)

(iii) $(A \cup B)' = A' \cap B'$ (De Morgan's law)

(iv) $(A \cap B)' = A' \cup B'$ (De Morgan's law)

(v) $(A')' = A$ (Law of complementation)

(vi) $\phi' = U$ (Law of empty set)

(vii) $U' = \phi$ and universal set)

Problems on Operation on Sets

Solved problems on operation on sets are given below to get a fair idea how to find the union and intersection of two or more sets.

We know, the union of sets is a set which contains all the elements in those sets and intersection of sets is a set which contains all the elements that are common in those sets.

Solved problems on operation on sets:

1. If $A = \{1, 3, 5\}$, $B = \{3, 5, 6\}$ and $C = \{1, 3, 7\}$

(i) Verify that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(ii) Verify $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Solution:

(i) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

$$\text{L.H.S.} = A \cup (B \cap C)$$

$$B \cap C = \{3\}$$

$$A \cup (B \cap C) = \{1, 3, 5\} \cup \{3\} = \{1, 3, 5\} \dots\dots\dots (1)$$

$$\text{R.H.S.} = (A \cup B) \cap (A \cup C)$$

$$A \cup B = \{1, 3, 5, 6\}$$

$$A \cup C = \{1, 3, 5, 7\}$$

$$(A \cup B) \cap (A \cup C) = \{1, 3, 5, 6\} \cap \{1, 3, 5, 7\} = \{1, 3, 5\} \dots\dots\dots (2)$$

From (1) and (2), we conclude that;

$$A \cup (B \cap C) = A \cup B \cap (A \cup C) \text{ [verified]}$$

(ii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

$$\text{L.H.S.} = A \cap (B \cup C)$$

$$B \cup C = \{1, 3, 5, 6, 7\}$$

$$A \cap (B \cup C) = \{1, 3, 5\} \cap \{1, 3, 5, 6, 7\} = \{1, 3, 5\} \dots\dots\dots (1)$$

$$\text{R.H.S.} = (A \cap B) \cup (A \cap C)$$

$$A \cap B = \{3, 5\}$$

$$A \cap C = \{1, 3\}$$

$$(A \cap B) \cup (A \cap C) = \{3, 5\} \cup \{1, 3\} = \{1, 3, 5\} \dots\dots\dots (2)$$

From (1) and (2), we conclude that;

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C) \text{ [verified]}$$

More worked-out problems on operation on sets to find the union and intersection of three sets.

2. Let $A = \{a, b, d, e\}$, $B = \{b, c, e, f\}$ and $C = \{d, e, f, g\}$

(i) Verify $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(ii) Verify $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Solution:

(i) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

$$\text{L.H.S.} = A \cap (B \cup C)$$

$$B \cup C = \{b, c, d, e, f, g\}$$

$$A \cap (B \cup C) = \{b, d, e\} \quad \dots\dots\dots (1)$$

$$\text{R.H.S.} = (A \cap B) \cup (A \cap C)$$

$$A \cap B = \{b, e\}$$

$$A \cap C = \{d, e\}$$

$$(A \cap B) \cup (A \cap C) = \{b, d, e\} \quad \dots\dots\dots (2)$$

From (1) and (2), we conclude that;

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C) \text{ [verified]}$$

(ii) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

$$\text{L.H.S.} = A \cup (B \cap C)$$

$$B \cap C = \{e, f\}$$

$$A \cup (B \cap C) = \{a, b, d, e, f\} \quad \dots\dots\dots (1)$$

$$\text{R.H.S.} = (A \cup B) \cap (A \cup C)$$

$$A \cup B = \{a, b, c, d, e, f\}$$

$$A \cup C = \{a, b, d, e, f, g\}$$

$$(A \cup B) \cap (A \cup C) = \{a, b, d, e, f\} \quad \dots\dots\dots (2)$$

From (1) and (2), we conclude that;

$$A \cup (B \cap C) = A \cup B \cap (A \cup C) \text{ [verified]}$$

Word Problems on Sets

Word problems on sets are solved here to get the basic ideas how to use the properties of union and intersection of sets.

Solved basic word problems on sets:

1. Let A and B be two finite sets such that $n(A) = 20$, $n(B) = 28$ and $n(A \cup B) = 36$, find $n(A \cap B)$.

Solution:

Using the formula $n(A \cup B) = n(A) + n(B) - n(A \cap B)$.

then $n(A \cap B) = n(A) + n(B) - n(A \cup B)$

$$= 20 + 28 - 36$$

$$= 48 - 36$$

$$= 12$$

2. If $n(A - B) = 18$, $n(A \cup B) = 70$ and $n(A \cap B) = 25$, then find $n(B)$.

Solution:

Using the formula $n(A \cup B) = n(A - B) + n(A \cap B) + n(B - A)$

$$70 = 18 + 25 + n(B - A)$$

$$70 = 43 + n(B - A)$$

$$n(B - A) = 70 - 43$$

$$n(B - A) = 27$$

Now $n(B) = n(A \cap B) + n(B - A)$

$$= 25 + 27$$

$$= 52$$

Different types on word problems on sets:

3. In a group of 60 people, 27 like cold drinks and 42 like hot drinks and each person likes at least one of the two drinks. How many like both coffee and tea?

Solution:

Let A = Set of people who like cold drinks.

B = Set of people who like hot drinks.

Given

$(A \cup B) = 60$ $n(A) = 27$ $n(B) = 42$ then;

$n(A \cap B) = n(A) + n(B) - n(A \cup B)$

$$= 27 + 42 - 60$$

$$= 69 - 60 = 9$$

$$= 9$$

Therefore, 9 people like both tea and coffee.

4. There are 35 students in art class and 57 students in dance class. Find the number of students who are either in art class or in dance class.

- When two classes meet at different hours and 12 students are enrolled in both activities.
- When two classes meet at the same hour.

Solution:

$$n(A) = 35, \quad n(B) = 57, \quad n(A \cap B) = 12$$

(Let A be the set of students in art class.
B be the set of students in dance class.)

(i) When 2 classes meet at different hours $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$$= 35 + 57 - 12$$

$$= 92 - 12$$

$$= 80$$

(ii) When two classes meet at the same hour, $A \cap B = \emptyset$ $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$$= n(A) + n(B)$$

$$= 35 + 57$$

$$= 92$$

Further concept to solve word problems on sets:

5. In a group of 100 persons, 72 people can speak English and 43 can speak French. How many can speak English only? How many can speak French only and how many can speak both English and French?

Solution:

Let A be the set of people who speak English.

B be the set of people who speak French.

A - B be the set of people who speak English and not French.

B - A be the set of people who speak French and not English.

$A \cap B$ be the set of people who speak both French and English.

Given,

$$n(A) = 72 \quad n(B) = 43 \quad n(A \cup B) = 100$$

Now, $n(A \cap B) = n(A) + n(B) - n(A \cup B)$

$$= 72 + 43 - 100$$

$$= 115 - 100$$

$$= 15$$

Therefore, Number of persons who speak both French and English = 15

$$n(A) = n(A - B) + n(A \cap B)$$

$$\Rightarrow n(A - B) = n(A) - n(A \cap B)$$

$$= 72 - 15$$

$$= 57$$

$$\text{and } n(B - A) = n(B) - n(A \cap B)$$

$$= 43 - 15$$

$$= 28$$

Therefore, Number of people speaking English only = 57

Number of people speaking French only = 28

Word problems on sets using the different properties (Union & Intersection):

6. In a competition, a school awarded medals in different categories. 36 medals in dance, 12 medals in dramatics and 18 medals in music. If these medals went to a total of 45 persons and only 4 persons got medals in all the three categories, how many received medals in exactly two of these categories?

Solution:

Let A = set of persons who got medals in dance.

B = set of persons who got medals in dramatics.

C = set of persons who got medals in music.

Given,

$$n(A) = 36 \qquad n(B) = 12 \qquad n(C) = 18$$

$$n(A \cup B \cup C) = 45 \qquad n(A \cap B \cap C) = 4$$

We know that number of elements belonging to exactly two of the three sets A, B, C

$$= n(A \cap B) + n(B \cap C) + n(A \cap C) - 3n(A \cap B \cap C)$$

$$= n(A \cap B) + n(B \cap C) + n(A \cap C) - 3 \times 4 \quad \dots\dots\dots(i)$$

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$$

$$\text{Therefore, } n(A \cap B) + n(B \cap C) + n(A \cap C) = n(A) + n(B) + n(C) + n(A \cap B \cap C) - n(A \cup B \cup C)$$

From (i) required number

$$= n(A) + n(B) + n(C) + n(A \cap B \cap C) - n(A \cup B \cup C) - 12$$

$$= 36 + 12 + 18 + 4 - 45 - 12$$

$$= 70 - 57$$

$$= 13$$

Apply set operations to solve the word problems on sets:

7. Each student in a class of 40 plays at least one indoor game chess, carrom and scrabble. 18 play chess, 20 play scrabble and 27 play carrom. 7 play chess and scrabble, 12 play scrabble and carrom and 4 play chess, carrom and scrabble. Find the number of students who play (i) chess and carrom. (ii) chess, carrom but not scrabble.

Solution:

Let A be the set of students who play chess

B be the set of students who play scrabble

C be the set of students who play carrom

Therefore, We are given $n(A \cup B \cup C) = 40$,

$$n(A) = 18, \quad n(B) = 20 \quad n(C) = 27,$$

$$n(A \cap B) = 7, \quad n(C \cap B) = 12 \quad n(A \cap B \cap C) = 4$$

We have

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$$

$$\text{Therefore, } 40 = 18 + 20 + 27 - 7 - 12 - n(C \cap A) + 4$$

$$40 = 69 - 19 - n(C \cap A)$$

$$40 = 50 - n(C \cap A) \quad n(C \cap A) = 50 - 40$$

$$n(C \cap A) = 10$$

Therefore, Number of students who play chess and carrom are 10.

Also, number of students who play chess, carrom and not scrabble.

$$= n(C \cap A) - n(A \cap B \cap C)$$

$$= 10 - 4$$

$$= 6$$

Therefore, we learned how to solve different types of word problems on sets without using Venn diagram.

Venn Diagrams in Different Situations

To draw Venn diagrams in different situations are discussed below:

How to represent a set using Venn diagrams in different situations?

1. ξ is a universal set and A is a subset of the universal set.

$$\xi = \{1, 2, 3, 4\}$$

$$A = \{2, 3\}$$

- Draw a rectangle which represents the universal set.
- Draw a circle inside the rectangle which represents A.
- Write the elements of A inside the circle.
- Write the leftover elements in ξ that is outside the circle but inside the rectangle.
- Shaded portion represents A' , i.e., $A' = \{1, 4\}$

2. ξ is a universal set. A and B are two disjoint sets but the subset of the universal set i.e., $A \subseteq \xi$, $B \subseteq \xi$ and $A \cap B = \phi$

For example;

$$\xi = \{a, e, i, o, u\}$$

$$A = \{a, i\}$$

$$B = \{e, u\}$$

- Draw a rectangle which represents the universal set.
- Draw two circles inside the rectangle which represents A and B.
- The circles do not overlap.
- Write the elements of A inside the circle A and the elements of B inside the circle B of ξ .
- Write the leftover elements in ξ , i.e., outside both circles but inside the rectangle.
- The figure represents $A \cap B = \phi$

3. ξ is a universal set. A and B are subsets of ξ . They are also overlapping sets.

For example;

$$\text{Let } \xi = \{1, 2, 3, 4, 5, 6, 7\}$$

$$A = \{2, 4, 6, 5\} \text{ and } B = \{1, 2, 3, 5\}$$

$$\text{Then } A \cap B = \{2, 5\}$$

- Draw a rectangle which represents a universal set.
- Draw two circles inside the rectangle which represents A and B.

- The circles overlap.
- Write the elements of A and B in the respective circles such that common elements are written in overlapping portion (2, 5).
- Write rest of the elements in the rectangle but outside the two circles.
- The figure represents $A \cap B = \{2, 5\}$

4. ξ is a universal set and A and B are two sets such that A is a subset of B and B is a subset of ξ .

For example;

Let $\xi = \{1, 3, 5, 7, 9\}$

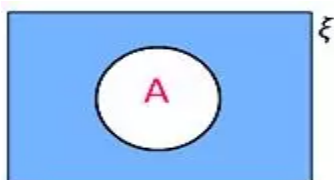
$A = \{3, 5\}$ and $B = \{1, 3, 5\}$

Then $A \subseteq B$ and $B \subseteq \xi$

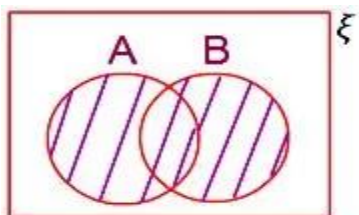
- Draw a rectangle which represents the universal set.
- Draw two circles such that circle A is inside circle B as $A \subseteq B$.
- Write the elements of A in the innermost circle.
- Write the remaining elements of B outside the circle A but inside the circle B.
- The leftover elements of are written inside the rectangle but outside the two circles.

Observe the Venn diagrams. The shaded portion represents the following sets.

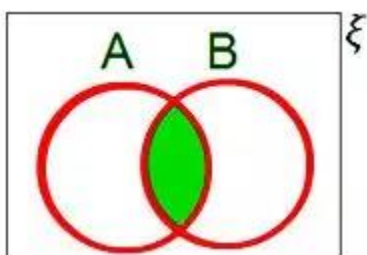
(a) A' (A dash)



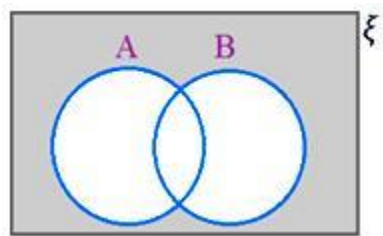
(b) $A \cup B$ (A union B)



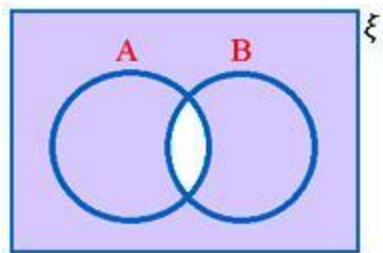
(c) $A \cap B$ (A intersection B)



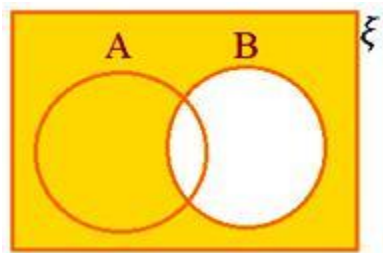
(d) $(A \cup B)'$ (A union B dash)



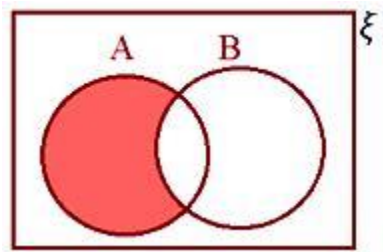
(e) $(A \cap B)'$ (A intersection B dash)



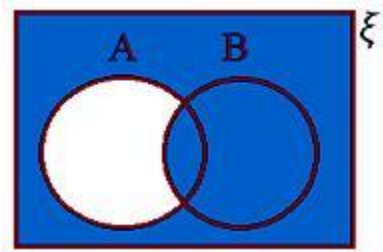
(f) B' (B dash)



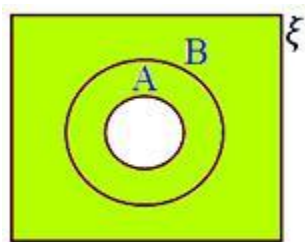
(g) $A - B$ (A minus B)



(h) $(A - B)'$ (Dash of sets A minus B)

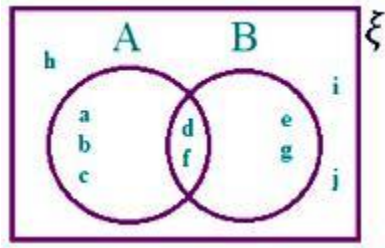


(i) $(A \subset B)'$ (Dash of A subset B)



For example;

Use Venn diagrams in different situations to find the following sets.



(a) $A \cup B$

(b) $A \cap B$

(c) A'

(d) $B - A$

(e) $(A \cap B)'$

(f) $(A \cup B)'$

Solution:

$$\xi = \{a, b, c, d, e, f, g, h, i, j\}$$

$$A = \{a, b, c, d, f\}$$

$$B = \{d, f, e, g\}$$

$$\mathbf{A \cup B} = \{\text{elements which are in A or in B or in both}\}$$

$$= \{a, b, c, d, e, f, g\}$$

$$\mathbf{A \cap B} = \{\text{elements which are common to both A and B}\}$$

$$= \{d, f\}$$

$$\mathbf{A'} = \{\text{elements of } \xi, \text{ which are not in A}\}$$

$$= \{e, g, h, i, j\}$$

$$\mathbf{B - A} = \{\text{elements which are in B but not in A}\}$$

$$= \{e, g\}$$

$$\mathbf{(A \cap B)'} = \{\text{elements of } \xi \text{ which are not in } A \cap B\}$$

$$= \{a, b, c, e, g, h, i, j\}$$

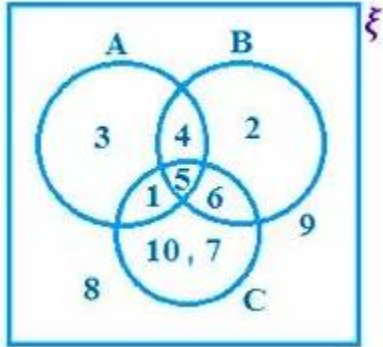
$$\mathbf{(A \cup B)'} = \{\text{elements of } \xi \text{ which are not in } A \cup B\}$$

$$= \{h, i, j\}$$

Examples on Venn Diagram

Solved examples on Venn diagram are discussed here.

From the adjoining Venn diagram, find the following sets.



- (i) **A**
- (ii) **B**
- (iii) **ξ**
- (iv) **A'**
- (v) **B'**
- (vi) **C'**
- (vii) **$C - A$**
- (viii) **$B - C$**
- (ix) **$A - B$**
- (x) **$A \cup B$**
- (xi) **$B \cup C$**
- (xii) **$A \cap C$**
- (xiii) **$B \cap C$**
- (xiv) **$(B \cup C)'$**
- (xv) **$(A \cap B)'$**
- (xvi) **$(A \cup B) \cap C$**
- (xvii) **$A \cap (B \cap C)$**

Answers for examples on Venn diagram are given below:

(i) **A**
 $= \{1, 3, 4, 5\}$

(ii) **B**
 $= \{4, 5, 6, 2\}$

(iii) ξ

$$= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

(iv) A'

$$= \{2, 6, 7, 8, 9, 10\} \quad \text{all elements of universal set leaving the elements of set A.}$$

(v) B'

$$= \{1, 3, 7, 8, 9, 10\} \quad \text{all elements of universal set leaving the elements of set B.}$$

(vi) C' = To find

$$C = \{1, 5, 6, 7, 10\}$$

$$\text{Therefore, } C' = \{2, 3, 4, 8, 9\} \quad \text{all elements of universal set leaving the elements of set C.}$$

(vii) $C - A$

$$\text{Here } C = \{1, 5, 6, 7, 10\}$$

$$A = \{1, 3, 4, 5\}$$

$$\text{then } C - A = \{6, 7, 10\} \quad \text{excluding all elements of A from C.}$$

(viii) $B - C$

$$\text{Here } B = \{4, 5, 6, 2\}$$

$$C = \{1, 5, 6, 7, 10\}$$

$$B - C = \{4, 2\} \quad \text{excluding all elements of C from B.}$$

(ix) $B - A$

$$\text{Here } B = \{4, 5, 2\}$$

$$A = \{1, 3, 4, 5\}$$

$$B - A = \{6, 2\} \quad \text{excluding all elements of A from C.}$$

(x) $A \cup B$

$$\text{Here } A = \{1, 3, 4, 5\}$$

$$B = \{4, 5, 6, 2\}$$

$$A \cup B = \{1, 2, 3, 4, 5, 6\}$$

(xi) $B \cup C$

Here $B = \{4, 5, 6, 2\}$

$C = \{1, 5, 6, 7, 10\}$

$B \cup C = \{1, 2, 4, 5, 6, 7, 10\}$

(xii) $(B \cup C)'$

Since, $B \cup C = \{1, 2, 4, 5, 6, 7, 10\}$

Therefore, $(B \cup C)' = \{3, 8, 9\}$

(xiii) $(A \cap B)'$

$A = \{1, 3, 4, 5\}$

$B = \{4, 5, 6, 2\}$

$(A \cap B) = \{4, 5\}$

$(A \cap B)' = \{1, 2, 3, 6, 7, 8, 9, 10\}$

(xiv) $(A \cup B) \cap C$

$A = \{1, 2, 3, 4\}$

$B = \{4, 5, 6, 2\}$

$C = \{1, 5, 6, 7, 10\}$

$A \cup B = \{1, 2, 3, 4, 5, 6\}$

$(A \cup B) \cap C = \{1, 5, 6\}$

(xv) $A \cap (B \cap C)$

$A = \{1, 3, 4, 5\}$

$B = \{4, 5, 6, 2\}$

$C = \{1, 5, 6, 7, 10\}$

$B \cap C = \{5, 6\}$

$A \cap (B \cap C) = \{5\}$

Difference of Two Sets

How to find the difference of two sets?

If A and B are two sets, then their difference is given by $A - B$ or $B - A$.

- If $A = \{2, 3, 4\}$ and $B = \{4, 5, 6\}$

$A - B$ means elements of A which are not the elements of B.

i.e., in the above example $A - B = \{2, 3\}$

In general, $B - A = \{x : x \in B, \text{ and } x \notin A\}$

- If A and B are disjoint sets, then $A - B = A$ and $B - A = B$

Solved examples to find the difference of two sets:

1. $A = \{1, 2, 3\}$ and $B = \{4, 5, 6\}$.

Find the difference between the two sets:

(i) A and B

(ii) B and A

Solution:

The two sets are disjoint as they do not have any elements in common.

(i) $A - B = \{1, 2, 3\} = A$

(ii) $B - A = \{4, 5, 6\} = B$

2. Let $A = \{a, b, c, d, e, f\}$ and $B = \{b, d, f, g\}$.

Find the difference between the two sets:

(i) A and B

(ii) B and A

Solution:

(i) $A - B = \{a, c, e\}$

Therefore, the elements a, c, e belong to A but not to B

(ii) $B - A = \{g\}$

Therefore, the element g belongs to B but not A.

3. Given three sets P, Q and R such that:

$P = \{x : x \text{ is a natural number between } 10 \text{ and } 16\}$,

$Q = \{y : y \text{ is a even number between } 8 \text{ and } 20\}$ and

$R = \{7, 9, 11, 14, 18, 20\}$

(i) Find the difference of two sets P and Q

(ii) Find $Q - R$

(iii) Find $R - P$

(iv) Find $Q - P$

Solution:

According to the given statements:

$$P = \{11, 12, 13, 14, 15\}$$

$$Q = \{10, 12, 14, 16, 18\}$$

$$R = \{7, 9, 11, 14, 18, 20\}$$

(i) $P - Q = \{\text{Those elements of set P which are not in set Q}\}$

$$= \{11, 13, 15\}$$

(ii) $Q - R = \{\text{Those elements of set Q not belonging to set R}\}$

$$= \{10, 12, 16\}$$

(iii) $R - P = \{\text{Those elements of set R which are not in set P}\}$

$$= \{7, 9, 18, 20\}$$

(iv) $Q - P = \{\text{Those elements of set Q not belonging to set P}\}$

$$= \{10, 16, 18\}$$

Complement of a Set

In complement of a set if ξ be the universal set and A a subset of ξ , then the complement of A is the set of all elements of ξ which are not the elements of A .

Symbolically, we denote the complement of A with respect to ξ as A' .

For Example; If $\xi = \{1, 2, 3, 4, 5, 6, 7\}$

$A = \{1, 3, 7\}$ find A' .

Solution:

We observe that 2, 4, 5, 6 are the only elements of ξ which do not belong to A .

Therefore, $A' = \{2, 4, 5, 6\}$

Note:

The complement of a universal set is an empty set.

The complement of an empty set is a universal set.

The set and its complement are disjoint sets.

For Example;

1. Let the set of natural numbers be the universal set and A is a set of even natural numbers,

then $A' = \{x: x \text{ is a set of odd natural numbers}\}$

2. Let $\xi =$ The set of letters in the English alphabet.

$A =$ The set of consonants in the English alphabet

then $A' =$ The set of vowels in the English alphabet.

3. Show that;

(a) The complement of a universal set is an empty set.

Let ξ denote the universal set, then

$\xi' =$ The set of those elements which are not in ξ .

$=$ empty set $= \phi$

Therefore, $\xi' = \phi$ so the complement of a universal set is an empty set.

(b) A set and its complement are disjoint sets.

Let A be any set then $A' =$ set of those elements of ξ which are not in A .

Let $x \notin A$, then x is an element of ξ not contained in A

So $x \notin A'$

Therefore, A and A' are disjoint sets.

Therefore, Set and its complement are disjoint sets

Similarly, in complement of a set when U be the universal set and A is a subset of U . Then the complement of A is the set all elements of U which are not the elements of A .

Symbolically, we write A' to denote the complement of A with respect to U .

Thus, $A' = \{x : x \in U \text{ and } x \notin A\}$

Obviously $A' = \{U - A\}$

For Example; Let $U = \{2, 4, 6, 8, 10, 12, 14, 16\}$

$A = \{6, 10, 4, 16\}$

$A' = \{2, 8, 12, 14\}$

We observe that 2, 8, 12, 14 are the only elements of U which do not belong to A .

Some properties of complement sets

(i) $A \cup A' = A' \cup A = U$ (Complement law)

(ii) $(A \cap B)' = \phi$ (Complement law)

(iii) $(A \cup B)' = A' \cap B'$ (De Morgan's law)

(iv) $(A \cap B)' = A' \cup B'$ (De Morgan's law)

(v) $(A')' = A$ (Law of complementation)

(vi) $\phi' = U$ (Law of empty set)

(vii) $U' = \phi$ and universal set)

