



EEE 1105: Electric Current and Ohm's Law

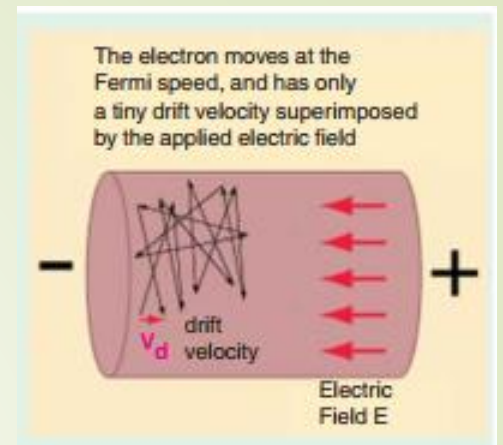
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1.1. Electron Drift Velocity



- Suppose that in a conductor, the number of free electrons available per m³ of the conductor material is **n**
- Let their axial drift velocity be **v** metres/second.
- In time **dt** , distance travelled would be **$v \times dt$** .
- If **A** is area of cross-section of the conductor, then the volume is **$vAdt$**
- the number of electrons contained in this volume is **$nvA dt$** .
- Obviously, all these electrons will cross the conductor cross-section in time **dt** .
- If **e** is the charge of each electron, then total charge which crosses the section in time dt is **$dq = nAev dt$** .
- Since current is the rate of flow of charge, it is given as

$$i = \frac{dq}{dt} = \frac{nAev dt}{dt} \quad \therefore i = nAev$$

Current density, $J = i/A = ne v$ ampere/metre²

1.4. Resistance

It may be defined as the property of a substance due to which it opposes (or restricts) the flow of electricity (i.e., electrons) through it.

1.5. The Unit of Resistance

The practical unit of resistance is ohm. A conductor is said to have a resistance of one ohm if it permits one ampere current to flow through it when one volt is impressed across its terminals.

1.7. Units of Resistivity

From Eq. (i), we have

$$\rho = \frac{AR}{l}$$

In the S.I. system of units,

$$\rho = \frac{A \text{ metre}^2 \times R \text{ ohm}}{l \text{ metre}} = \frac{AR}{l} \text{ ohm-metre}$$

Hence, the unit of resistivity is ohm-metre ($\Omega\text{-m}$).

1.6. Laws of Resistance

The resistance R offered by a conductor depends on the following factors :

- (i) It varies directly as its length, l .
- (ii) It varies inversely as the cross-section A of the conductor.
- (iii) It depends on the nature of the material.
- (iv) It also depends on the temperature of the conductor.

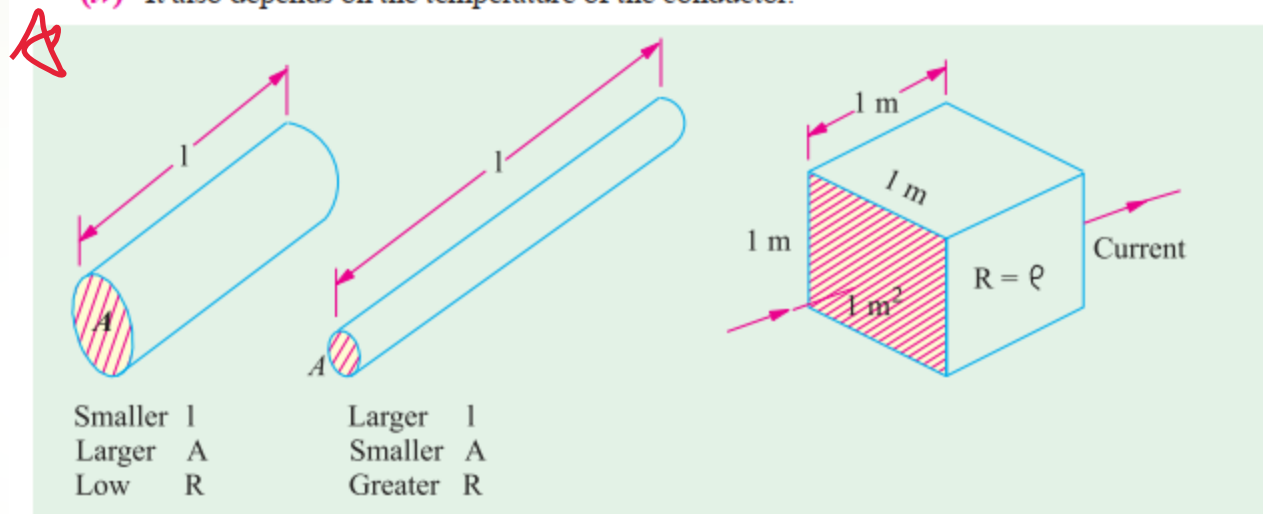


Fig. 1.3.

Fig. 1.4

Neglecting the last factor for the time being, we can say that

$$R \propto \frac{l}{A} \quad \text{or} \quad R = \rho \frac{l}{A} \quad \dots(i)$$

where ρ is a constant depending on the nature of the material of the conductor and is known as its **specific resistance or resistivity**.

If in Eq. (i), we put

$$l = 1 \text{ metre} \quad \text{and} \quad A = 1 \text{ metre}^2, \text{ then } R = \rho \quad (\text{Fig. 1.4})$$

Hence, specific resistance of a material may be defined as **the resistance between the opposite faces of a metre cube of that material**.

1.8. Conductance and Conductivity

Conductance (G) is reciprocal of resistance*. Whereas resistance of a conductor measures the *opposition* which it offers to the flow of current, the conductance measures the *inducement* which it offers to its flow.

From Eq. (i) of Art. 1.6, $R = \rho \frac{l}{A}$ or $G = \frac{1}{\rho} \cdot \frac{A}{l} = \frac{\sigma A}{l}$

where σ is called the *conductivity or specific conductance* of a conductor. The unit of conductance is siemens (S). Earlier, this unit was called mho.

It is seen from the above equation that the conductivity of a material is given by

$$\sigma = G \frac{l}{A} = \frac{G \text{ siemens} \times l \text{ metre}}{A \text{ metre}^2} = G \frac{l}{A} \text{ siemens/metre}$$

Hence, the unit of conductivity is siemens/metre (S/m).

1.9. Effect of Temperature on Resistance

The effect of rise in temperature is :

- (i) to *increase* the resistance of pure metals. The increase is large and fairly regular for normal ranges of temperature. The temperature/resistance graph is a straight line (Fig. 1.6). As would be presently clarified, metals have a positive temperature co-efficient of resistance.
- (ii) to *increase* the resistance of alloys, though in their case, the increase is relatively small and irregular. For some high-resistance alloys like Eureka (60% Cu and 40% Ni) and manganin, the increase in resistance is (or can be made) negligible over a considerable range of temperature.
- (iii) to *decrease* the resistance of electrolytes, insulators (such as paper, rubber, glass, mica etc.) and partial conductors such as carbon. Hence, insulators are said to possess a *negative* temperature-coefficient of resistance.

1.13. Ohm's Law

The ratio of potential difference (V) between any two points on a conductor to the current (I) flowing between them, is constant, provided the temperature of the conductor does not change.

In other words,

$$\frac{V}{I} = \text{constant} \quad \text{or} \quad \frac{V}{I} = R$$

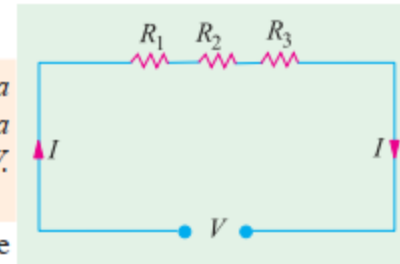
where R is the resistance of the conductor between the two points considered.



George Simon Ohm

Example 1.24. Three resistors are connected in series across a 12-V battery. The first resistor has a value of $1\ \Omega$, second has a voltage drop of 4 V and the third has a power dissipation of 12 W . Calculate the value of the circuit current.

Solution. Let the two unknown resistors be R_2 and R_3 and I the circuit current



$$\therefore I^2 R_3 = 12 \quad \text{and} \quad IR_3 = 4 \quad \therefore R_3 = \frac{3}{4} R_2^2. \quad \text{Also, } I = \frac{4}{R_2}$$

$$\text{Now, } I(1 + R_2 + R_3) = 12$$

Substituting the values of I and R_3 , we get

$$\frac{4}{R_2} \left(1 + R_2 + \frac{3}{4} R_2^2 \right) = 12 \quad \text{or} \quad 3R_2^2 - 8R_2 + 4 = 0$$

$$\therefore R_2 = \frac{8 \pm \sqrt{64 - 48}}{6} \quad \therefore R_2 = 2\ \Omega \quad \text{or} \quad \frac{2}{3}\ \Omega$$

$$\therefore R_3 = \frac{3}{4} R_2^2 = \frac{3}{4} \times 2^2 = 3\ \Omega \quad \text{or} \quad \frac{3}{4} \left(\frac{2}{3} \right)^2 = \frac{1}{3}\ \Omega$$

$$\therefore I = \frac{12}{1 + 2 + 3} = 2\text{ A} \quad \text{or} \quad I = \frac{12}{1 + (2/3) + (1/3)} = 6\text{ A}$$

1.14. Resistance in Series

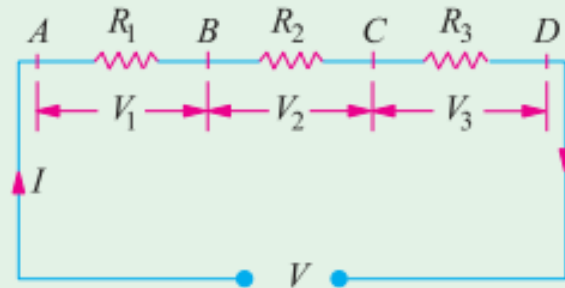


Fig. 1.12

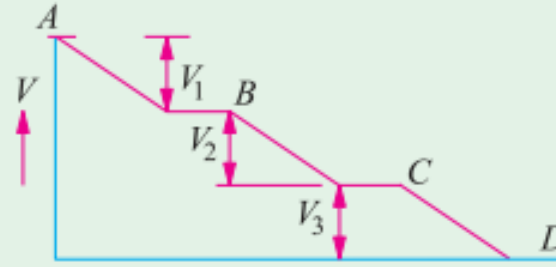


Fig. 1.13

The equivalent resistance or total resistance between points A and D is equal to the sum of the three individual resistances.

remembered that

- (i) current is the same through all the three conductors
- (ii) but voltage drop across each is different due to its different resistance and is given by Ohm's Law
- (iii) sum of the three voltage drops is equal to the voltage applied across the three conductors

There is a progressive fall in potential as we go from point A to D as shown in Fig. 1.13

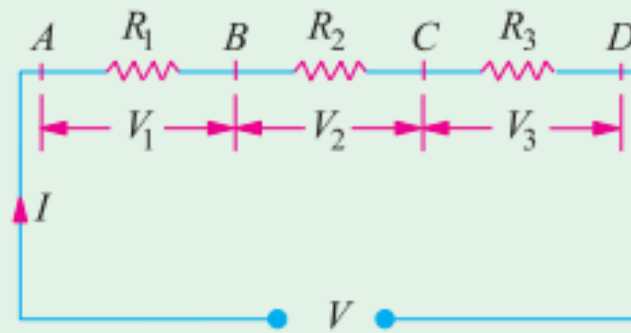


Fig. 1.12

$$\therefore V = V_1 + V_2 + V_3 = IR_1 + IR_2 + IR_3$$

—Ohm's Law

But

$$V = IR$$

where R is the equivalent resistance of the series combination.

$$\therefore IR = IR_1 + IR_2 + IR_3 \quad \text{or} \quad R = R_1 + R_2 + R_3$$

Also

$$\frac{1}{G} = \frac{1}{G_1} + \frac{1}{G_2} + \frac{1}{G_3}$$

As seen from above, the main characteristics of a series circuit are :

1. same current flows through all parts of the circuit.
2. different resistors have their individual voltage drops.
3. voltage drops are additive.
4. applied voltage equals the sum of different voltage drops.
5. resistances are additive.
6. powers are additive.

1.15. Voltage Divider Rule

Since in a series circuit, same current flows through each of the given resistors, voltage drop varies directly with its resistance. In Fig. 1.14 is shown a 24-V battery connected across a series combination of three resistors.

$$\text{Total resistance} \quad R = R_1 + R_2 + R_3 = 12 \, \Omega$$

According to Voltage Divider Rule, various voltage drops are :

$$V_1 = V \cdot \frac{R_1}{R} = 24 \times \frac{2}{12} = 4 \, \text{V}$$

$$V_2 = V \cdot \frac{R_2}{R} = 24 \times \frac{4}{12} = 8 \, \text{V}$$

$$V_3 = V \cdot \frac{R_3}{R} = 24 \times \frac{6}{12} = 12 \, \text{V}$$

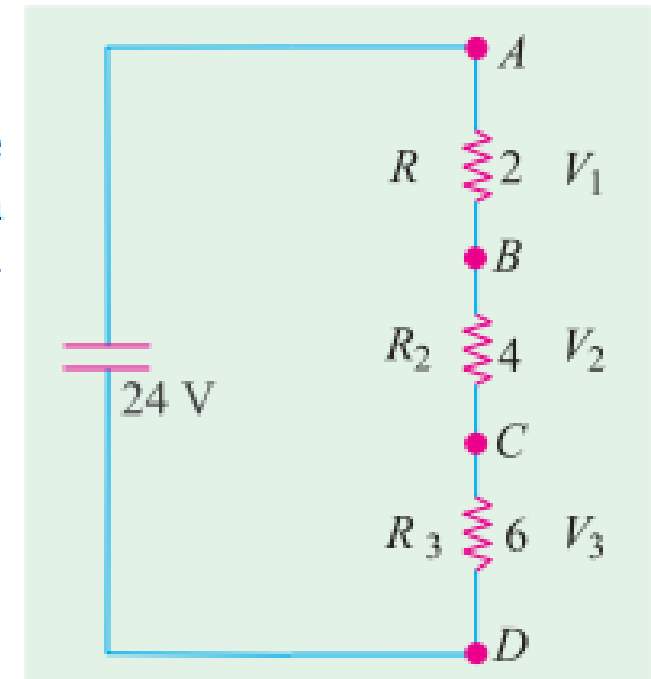


Fig.1.14

1.16. Resistances in Parallel

Three resistances, as joined in Fig. 1.15 are said to be connected in parallel. In this case (i) p.d. across all resistances is the same (ii) current in each resistor is different and is given by Ohm's Law and (iii) the total current is the sum of the three separate currents.

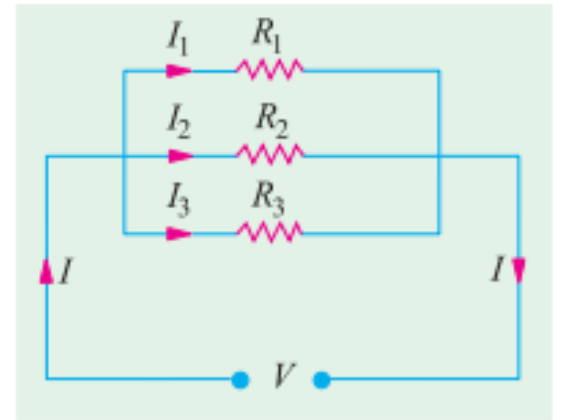


Fig.1.15

$$I = I_1 + I_2 + I_3 = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

Now,

$$I = \frac{V}{R} \text{ where } V \text{ is the applied voltage.}$$

R = equivalent resistance of the parallel combination.

$\therefore$

$$\frac{V}{R} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} \quad \text{or} \quad \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Also

$$G = G_1 + G_2 + G_3$$

The main characteristics of a parallel circuit are :

1. same voltage acts across all parts of the circuit
2. different resistors have their individual current.
3. branch currents are additive.
4. conductances are additive.
5. powers are additive.

Example 1.25. What is the value of the unknown resistor R in Fig. 1.16 if the voltage drop across the $500\ \Omega$ resistor is 2.5 volts? All resistances are in ohm. (Elect. Technology, Indore Univ.)

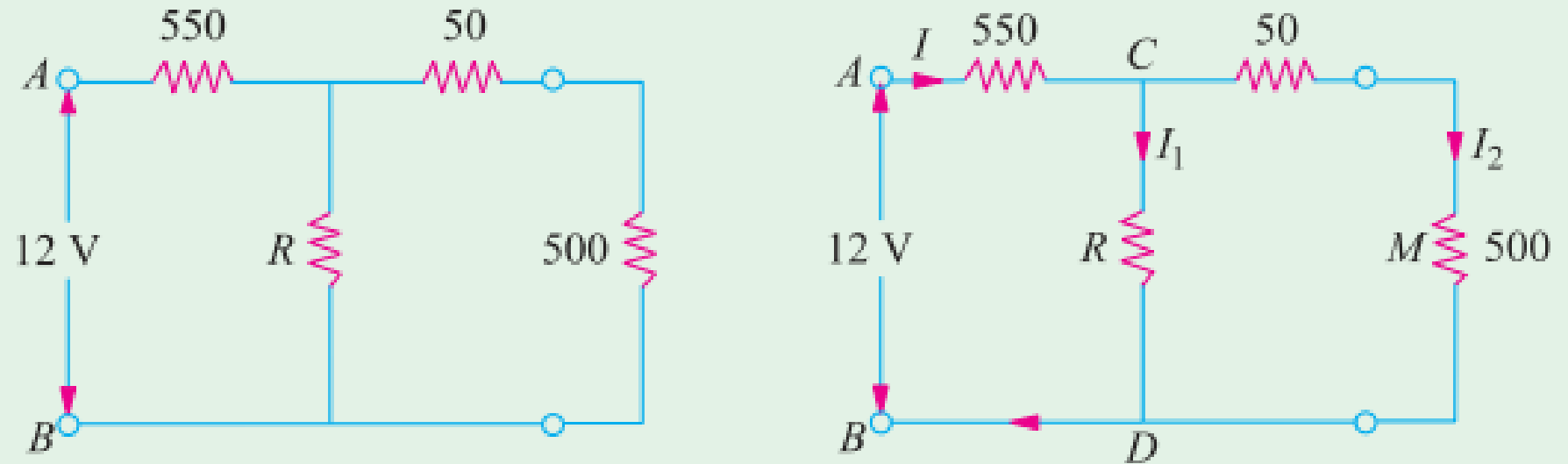


Fig. 1.16

Solution. By direct proportion, drop on $50\ \Omega$ resistance $= 2.5 \times 50/500 = 0.25\ \text{V}$

Drop across CMD or CD $= 2.5 + 0.25 = 2.75\ \text{V}$

Drop across $550\ \Omega$ resistance $= 12 - 2.75 = 9.25\ \text{V}$

$$I = 9.25/550 = 0.0168\ \text{A}, I_2 = 2.5/500 = 0.005\ \text{A}$$

$$I_1 = 0.0168 - 0.005 = 0.0118\ \text{A}$$

$$\therefore 0.0118 = 2.75/R; \quad R = \mathbf{233\ \Omega}$$

1.20. Short and Open Circuits

When two points of circuit are connected together by a thick metallic wire (Fig. 1.38), they are said to be *short-circuited*. Since 'short' has practically zero resistance, it gives rise to two important facts :

- (i) no voltage can exist across it because $V = IR = I \times 0 = 0$
- (ii) current through it (called short-circuit current) is very large (theoretically, infinity)

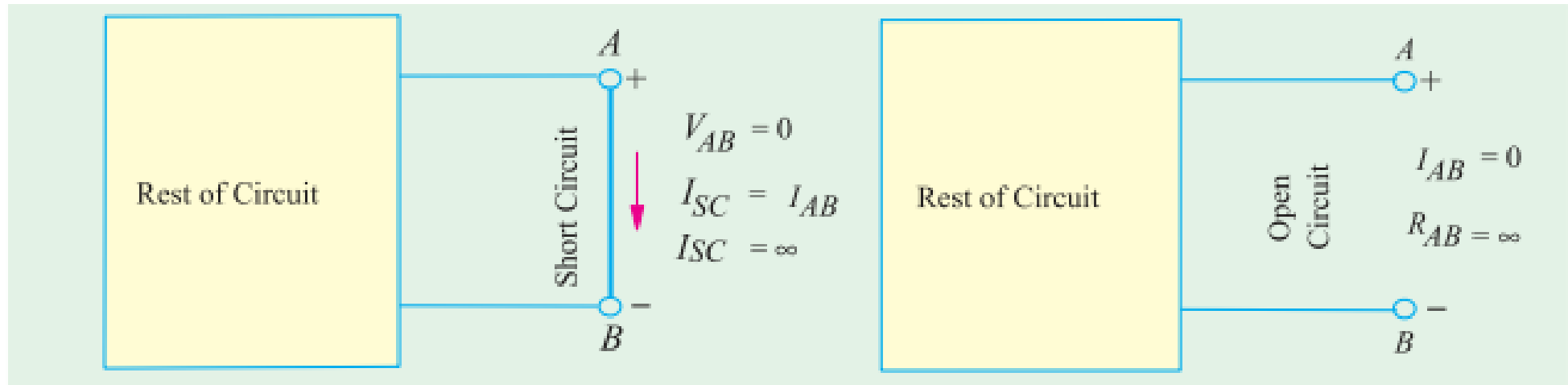


Fig. 1.38

Fig. 1.39

Two points are said to be open-circuited when there is no direct connection between them (Fig. 1.39). Obviously, an 'open' represents a break in the continuity of the circuit. Due to this break

- (i) resistance between the two points is infinite.
- (ii) there is no flow of current between the two points.

1.21. 'Shorts' in a Series Circuit

Since a dead (or solid) short has almost zero resistance, it causes the problem of excessive current which, in turn, causes power dissipation to increase many times and circuit components to burn out.

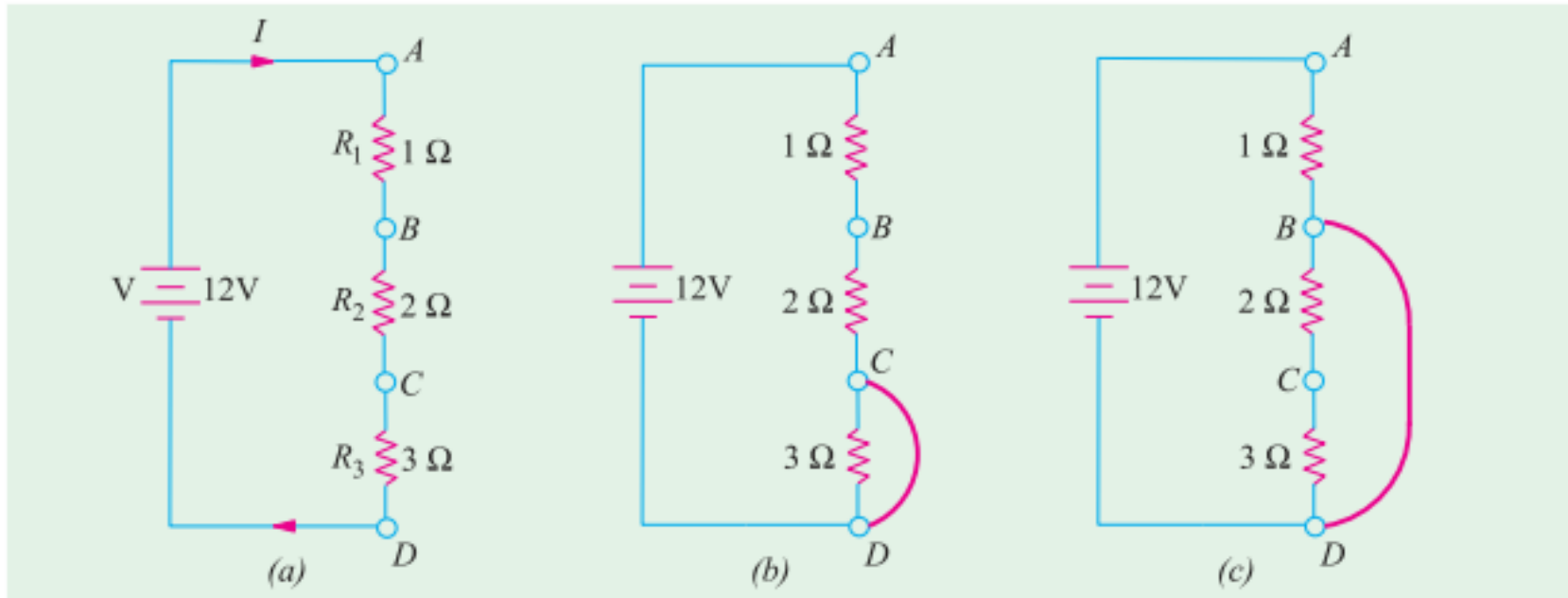


Fig. 1.40

In Fig. 1.40 (a) is shown a normal series circuit where

$$V = 12 \text{ V}, R = R_1 + R_2 + R_3 = 6 \Omega$$

$$I = V/R = 12/6 = 2 \text{ A}, P = I^2 R = 2^2 \times 6 = 24 \text{ W}$$

In Fig. 1.40 (b), 3-Ω resistor has been shorted out by a resistanceless copper wire so that $R_{CD} = 0$. Now, total circuit resistance $R = 1 + 2 + 0 = 3 \Omega$. Hence, $I = 12/3 = 4 \text{ A}$ and $P = 4^2 \times 3 = 48 \text{ W}$.

Fig. 1.40 (c) shows the situation where both 2 Ω and 3 Ω resistors have been shorted out of the circuit. In this case,

$$R = 1 \Omega, I = 12/1 = 12 \text{ A} \quad \text{and} \quad P = 12^2 \times 1 = 144 \text{ W}$$

Because of this excessive current (6 times the normal value), connecting wires and other circuit components can become hot enough to ignite and burn out.

1.22. 'Opens' in a Series Circuit

In a normal series circuit like the one shown in Fig. 1.41 (a), there exists a current flow and the voltage drops across different resistors are proportional to their resistances. If the circuit becomes 'open' anywhere, following two effects are produced :

(i) since 'open' offers infinite resistance, circuit current becomes zero. Consequently, there is no voltage drop across R_1 and R_2 .

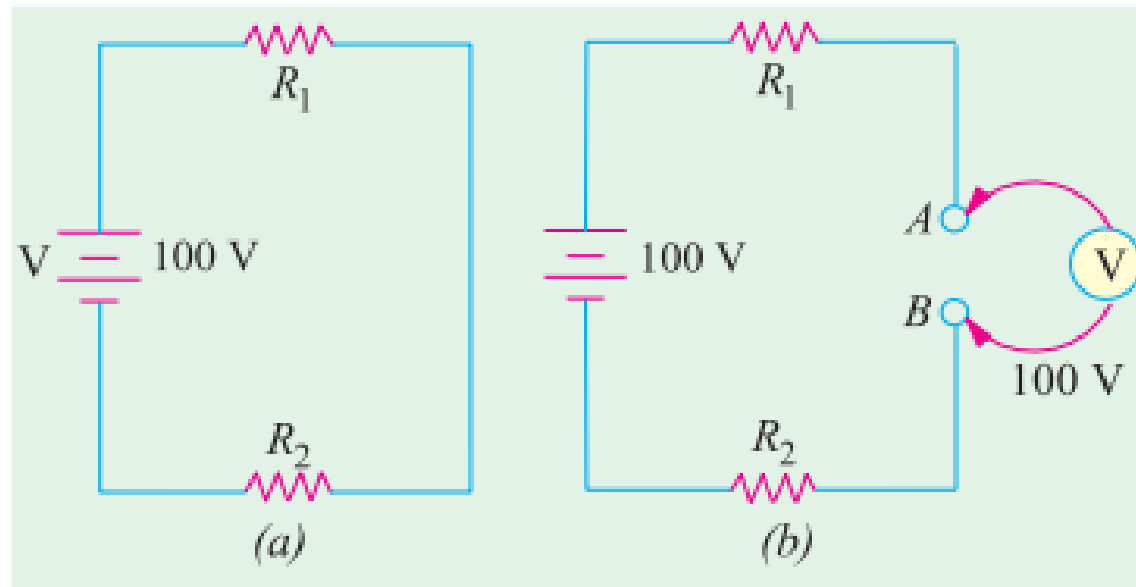


Fig. 1.41

(ii) *whole of the applied voltage (i.e. 100 V in this case) is felt across the 'open' i.e. across terminals A and B [Fig. 1.41 (b)].*

The reason for this is that R_1 and R_2 become negligible as compared to the infinite resistance of the 'open' which has practically whole of the applied voltage dropped across it (as per Voltage Divider Rule of art. 1.15). Hence, voltmeter in Fig. 1.41 (b) will read nearly 100 V i.e. the supply voltage.

1.23. 'Opens' in a Parallel Circuit

Since an 'open' offers infinite resistance, there would be no current in that part of the circuit where it occurs. In a parallel circuit, an 'open' can occur either in the main line or in any parallel branch.

As shown in Fig. 1.42 (a), an open in the main line prevents flow of current to *all branches*. Hence, neither of the two bulbs glows. However, full applied voltage (i.e. 220 V in this case) *is available across the open*.

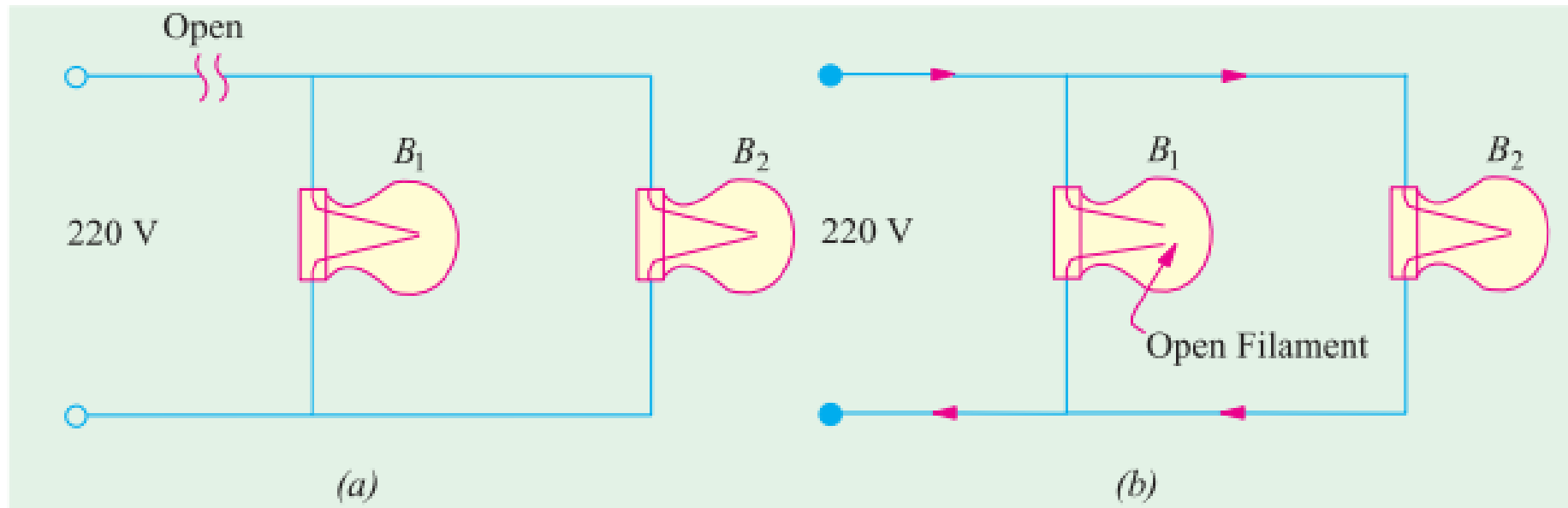


Fig. 1.42

In this Fig. 1.42 (b), 'open' has occurred in branch circuits of B_1 . Since there is no current in this branch, B_1 will not glow. However, as the other bulb remains connected across the voltage supply, it would keep operating normally.

It may be noted that if a voltmeter is connected across the open bulb, it will read full supply voltage of 220 V.

1.24. 'Shorts' in Parallel Circuits

Suppose a 'short' is placed across R_3 (Fig. 1.43). It becomes directly connected across the battery and draws almost infinite current because not only its own resistance but that of the connecting wires AC and BD is negligible. Due to this excessive current, the wires may get hot enough to burn out unless the circuit is protected by a fuse.

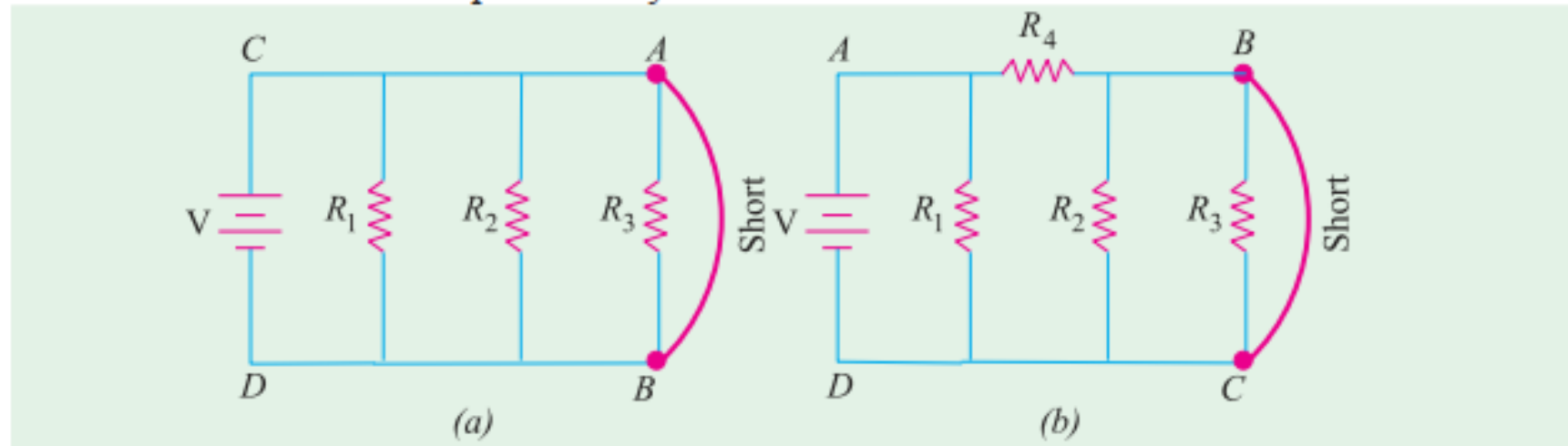


Fig. 1.43

Following points about the circuit of Fig. 1.43 (a) are worth noting.

1. not only is R_3 short-circuited but both R_1 and R_2 are also shorted out *i.e.* **short across one branch means short across all branches.**
2. there is no current in shorted resistors. If there were three bulbs, they will not glow.
3. the shorted components are not damaged, For example, if we had three bulbs in Fig. 1.43 (a), they would glow again when circuit is restored to normal conditions by removing the short-circuited.

It may, however, be noted from Fig. 1.43 (b) that a short-circuit across R_3 may short out R_2 but not R_1 since it is protected by R_4 .

1.25. Division of Current in Parallel Circuits

In Fig. 1.44, two resistances are joined in parallel across a voltage V . The current in each branch, as given in Ohm's law, is

$$I_1 = V/R_1 \text{ and } I_2 = V/R_2$$

$$\therefore \frac{I_1}{I_2} = \frac{R_2}{R_1}$$

$$\text{As } \frac{I}{R_1} = G_1 \text{ and } \frac{I}{R_2} = G_2$$

$$\therefore \frac{I_1}{I_2} = \frac{G_2}{G_1}$$

Hence, the division of current in the branches of a parallel circuit is directly proportional to the conductance of the branches or inversely proportional to their resistances. We may also express the branch currents in terms of the total circuit current thus :

$$\text{Now } I_1 + I_2 = I; \quad \therefore I_2 = I - I_1 \quad \therefore \frac{I_1}{I - I_1} = \frac{R_2}{R_1} \quad \text{or } I_1 R_1 = R_2 (I - I_1)$$

$$\therefore I_1 = I \frac{R_1}{R_1 + R_2} = I \frac{G_2}{G_1 + G_2} \quad \text{and } I_2 = I \frac{R_2}{R_1 + R_2} = I \frac{G_1}{G_1 + G_2}$$

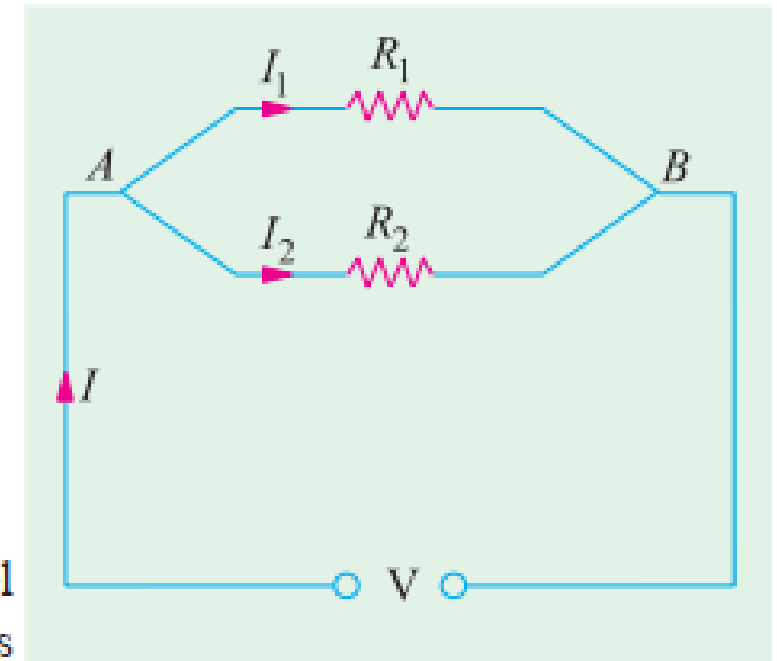


Fig. 1.44

This Current Divider Rule has direct application in solving electric circuits by Norton's theorem (Art. 2.25).

Take the case of three resistors in parallel connected across a voltage V (Fig. 1.45). Total current is $I = I_1 + I_2 + I_3$. Let the equivalent resistance be R . Then

$$V = IR$$

Also $V = I_1 R_1 \quad \therefore IR = I_1 R_1$

or $\frac{I}{I_1} = \frac{R_1}{R} \quad \text{or} \quad I_1 = IR/R_1 \quad \dots (i)$

Now $\frac{I}{R} = \frac{I}{R_1} + \frac{I}{R_2} + \frac{I}{R_3}$

$$R = \frac{R_1 R_2 R_3}{R_2 R_3 + R_2 R_3 + R_1 R_2}$$

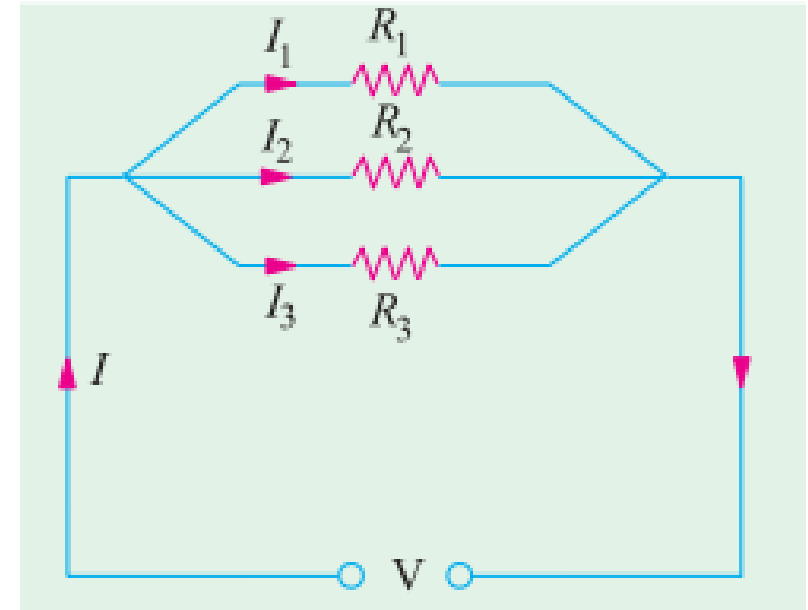


Fig. 1.45

From (i) above, $I_1 = I \left(\frac{R_2 R_3}{R_1 R_2 + R_2 R_3 + R_3 R_1} \right) = I \cdot \frac{G_1}{G_1 + G_2 + G_3}$

Similarly, $I_2 = I \frac{R_1 R_3}{R_1 R_2 + R_2 R_3 + R_3 R_1} = I \cdot \frac{G_2}{G_1 + G_2 + G_3}$

$$I_3 = I \frac{R_1 R_2}{R_1 R_2 + R_2 R_3 + R_3 R_1} = I \cdot \frac{G_3}{G_1 + G_2 + G_3}$$

Example 1.37. A resistance of $10\ \Omega$ is connected in series with two resistances each of $15\ \Omega$ arranged in parallel. What resistance must be shunted across this parallel combination so that the total current taken shall be 1.5 A with 20 V applied?

(Elements of Elect. Engg.-1; Bangalore Univ.)

Solution. The circuit connections are shown in Fig. 1.46.

Drop across $10\text{-}\Omega$ resistor $= 1.5 \times 10 = 15\text{ V}$

Drop across parallel combination, $V_{AB} = 20 - 15 = 5\text{ V}$

Hence, voltage across each parallel resistance is 5 V .

$$I_1 = 5/15 = 1/3\text{ A}, I_2 = 5/15 = 1/3\text{ A}$$

$$I_3 = 1.5 - (1/3 + 1/3) = 5/6\text{ A}$$

$$\therefore I_3 R = 5 \quad \text{or} \quad (5/6) R = 5 \quad \text{or} \quad R = 6\ \Omega$$

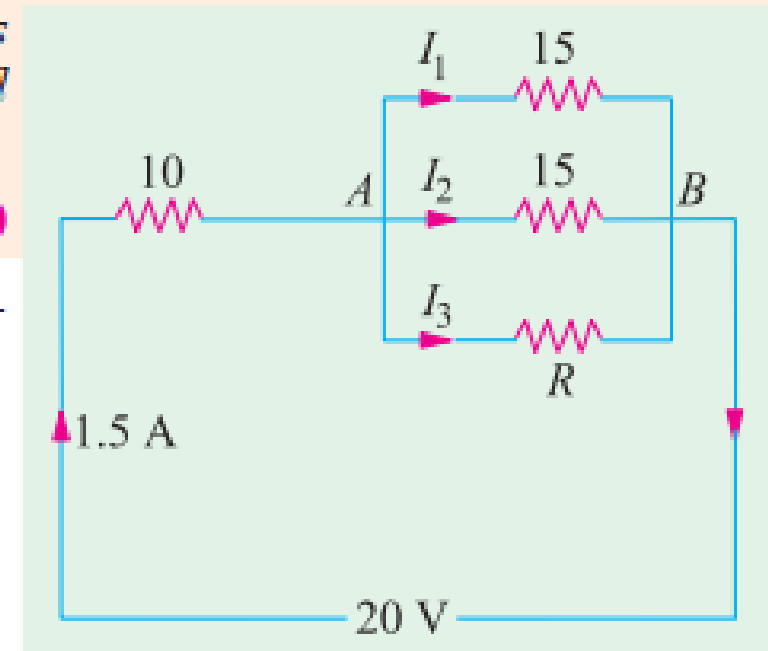


Fig. 1.46



1.26. Equivalent Resistance

- ▶ The equivalent resistance of a circuit (or network) between its any two points (or terminals) is given by that **single** resistance which can replace the entire given circuit between **these two points**.
- 

1.29. Voltage Divider Circuit

A voltage divider circuit (also called potential divider) is a series network which is used to feed other networks with a number of different voltages and derived from a single input voltage source.

Fig. 1.89 (a) shows a simple voltage divider circuit which provides two output voltages V_1 and V_2 . Since no load is connected across the output terminals, it is called an *unloaded* voltage divider.

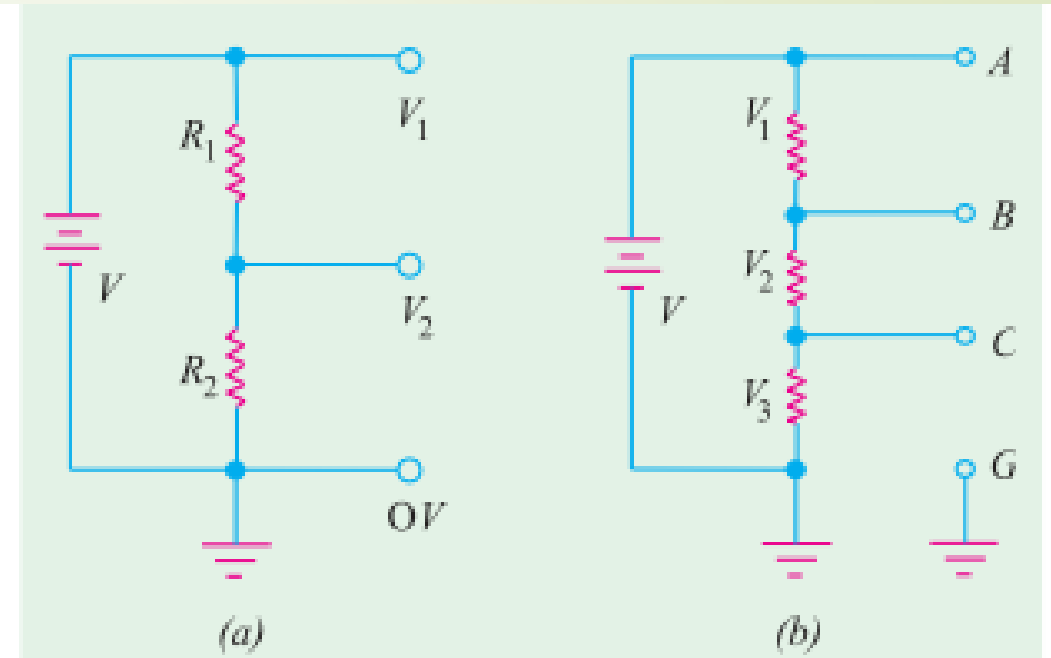


Fig. 1.89

As seen from Art. 1.15.

$$V_1 = V \frac{R_1}{R_1 + R_2} \text{ and } V_2 = V \cdot \frac{R_2}{R_1 + R_2}$$

The ratio V_2/V is also known as voltage-ratio *transfer function*.

$$\text{As seen, } \frac{V_2}{V} = \frac{R_2}{R_1 + R_2} = \frac{1}{1 + R_1/R_2}$$

The voltage divider of Fig. 1.89 (b) can be used to get six different voltages :

$$V_{CG} = V_3, V_{BC} = V_2, V_{AB} = V_1, V_{BG} = (V_2 + V_3), V_{AC} = (V_1 + V_2) \text{ and } V_{AG} = V$$

Example 1.51. Find the values of different voltages that can be obtained from a 12-V battery with the help of voltage divider circuit of Fig. 1.90.

Solution.

$$R = R_1 + R_2 + R_3 = 4 + 3 + 1 = 8 \, \Omega$$

Drop across

$$R_1 = 12 \times 4/8 = 6 \, \text{V}$$

$\therefore$

$$V_B = 12 - 6 = 6 \, \text{V above ground}$$

Drop across

$$R_2 = 12 \times 3/8 = 4.5 \, \text{V}$$

$\therefore$

$$V_C = V_B - 4.5 = 6 - 4.5 = 1.5$$

Drop across

$$R_3 = 12 \times 1/8 = 1.5 \, \text{V}$$

Different available load voltages are :

$$(i) \, V_{AB} = V_A - V_B = 12 - 6 = 6 \, \text{V}$$

$$(ii) \, V_{AC} = 12 - 1.5 = 10.5 \, \text{V}$$

$$(iv) \, V_{BC} = 6 - 1.5 = 4.5 \, \text{V}$$

$$(iii) \, V_{AD} = 12 \, \text{V}$$

$$(v) \, V_{CD} = 1.5 \, \text{V}$$

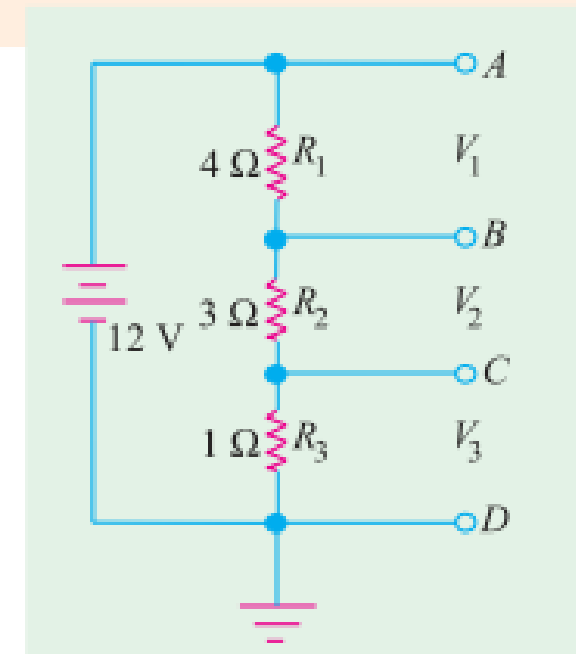


Fig. 1.90



Thanks

➤ Q & A

